This is an open-book, open-notes exam. You cannot use the computer.

You have 1 hour 15 minutes to work on this exam. The examination has 5 questions worth 70 points. There is a final bonus question, worth 5 points, which will NOT be curved. Please budget your time accordingly. Write your answers in the space provided on the exam. If you use additional scratch paper, please turn that in as well.

Your Name: ___________________________

In accordance with both the letter and spirit of the Honor Code, I have neither given nor received assistance on this examination.

Signature: ____________________________

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Score</th>
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<tbody>
<tr>
<td>1</td>
<td>15</td>
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<td>2</td>
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<td>5</td>
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<td>Total</td>
<td>70</td>
<td></td>
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<tr>
<td>Bonus</td>
<td>5</td>
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</table>
1. (15 points) True or false. Explain your answer. No credit is given unless the explanation is correct.

(a) Consider the DOM relation, where $a$ DOM $b$ is read “a node, $a$, dominates a node, $b$”. If $a$ DOM $c$ and $b$ DOM $c$, then either $a$ DOM $b$ or $b$ DOM $a$.

(b) Lazy code motion CANNOT move expressions out of a cycle in a control flow graph, if it is not part of a natural loop.

(c) Lazy code motion should be run before constant propagation.

(d) Given a monotone dataflow framework, an iterative algorithm will converge to a fixed point solution even if all the interior program points of a program are initialized to bottom.

(e) Given a monotone dataflow framework, if an input flow graph has no cycles, an iterative algorithm will produce the maximum fixed point solution even if all the interior program points are initialized to bottom.
2. (5 points) Suppose you have a register interference graph on a machine with 5 registers, where every node in the graph has 6 edges. Is this graph impossible to color (with 5 colors)?

Either provide an argument that such a graph cannot have a 5-coloring, or give an example of such a graph that does have a 5-coloring.

3. (10 points) Consider the following flow graph (A is the start node):

(a) Give all the natural loops of this graph.

(b) Is this graph reducible? If yes, explain. If no, give a minimal set of edges that should be added or removed to make it reducible.
4. (15 points) Show the result of applying partial redundancy elimination to the following program. Just show the result of the optimization — it is not necessary to show intermediate steps.
5. (25 points) Define a static program analysis that eliminates dead code from a program. Assume a programming language with simple integer variables and the following statements:

- input functions: input(x),
- output functions: output(x),
- x1 = x2;
- x1 = x2 + x3;
- if (x) goto L;

You cannot eliminate any input, output, or conditional branch instructions. Any assignments and additions that do not contribute to the computation of the variables used in output and conditional statements should be eliminated.

Given the following program:

```plaintext
input(a);
b = a;
c = a;
L1: c = b + c;
    input (a);
    if (a) goto L1;
d = a;
e = a + d;
    if (e) goto L2;
f = 1;
L2: output (e);
```

The following code should be generated:

```plaintext
input(a);
L1: input(a);
    if (a) goto L1;
d = a;
e = a + d;
    if (e) goto L2;
L2: output (e);
```

Specify your data flow algorithm fully by giving concise answers in the third column of the following table: (Note that you will not receive full credit if the algorithm does not necessarily converge). For the sake of simplicity, treat each statement as a separate basic block. Please state clearly how you eliminate the dead code after the data flow analysis.
<table>
<thead>
<tr>
<th></th>
<th>Direction</th>
<th>Forward or backward</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>Set of values in the semi-lattice</td>
<td>Explain what the values mean</td>
</tr>
<tr>
<td>c</td>
<td>Semi-lattice diagram</td>
<td>Label the top and the bottom elements</td>
</tr>
<tr>
<td>d</td>
<td>Transfer function of a basic block</td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>Boundary condition</td>
<td>Assignment to: out[entry] or in[exit]</td>
</tr>
<tr>
<td>f</td>
<td>Initialization for the iterative algorithm</td>
<td></td>
</tr>
<tr>
<td>g</td>
<td>Is the dataflow framework monotone?</td>
<td>Yes or no. No explanation is necessary.</td>
</tr>
<tr>
<td>h</td>
<td>Is the framework distributive?</td>
<td>Yes or no. No explanation is necessary.</td>
</tr>
<tr>
<td>i</td>
<td>Does the algorithm necessarily converge?</td>
<td>Yes or no. Explain why.</td>
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</tbody>
</table>
6. (Bonus 5 points) Describe briefly how you can improve the dead code elimination algorithm from Problem 5 if you are allowed to remove unnecessary conditional statements.