

Lecture 6

Register Allocation

- I. Introduction
- II. Abstraction and the Problem
- III. Algorithm

Reading: Chapter 8.8.4

Before next class: Chapter 10.1 - 10.2

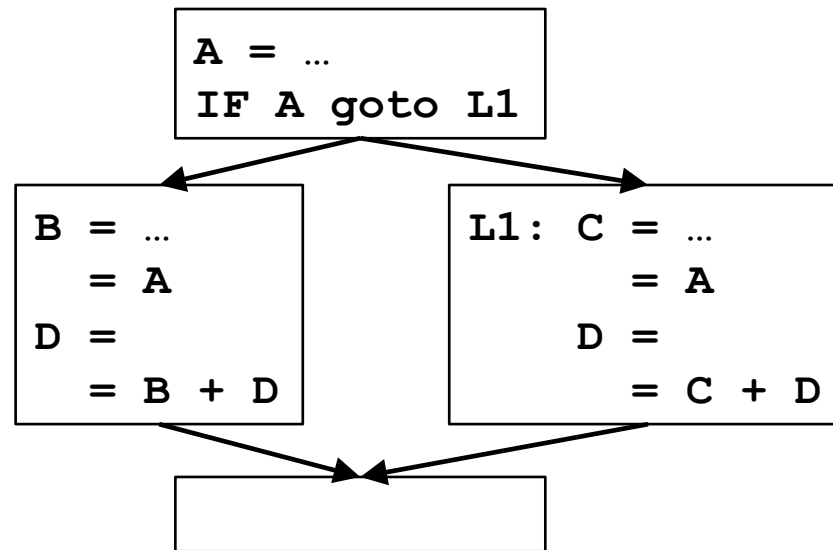
I. Motivation

- **Problem**
 - Allocation of variables (pseudo-registers) to hardware registers in a procedure
- **Perhaps the most important optimization**
 - Directly reduces running time
 - (memory access → register access)
 - Useful for other optimizations
 - e.g. cse assumes old values are kept in registers.

Goal

- Find an assignment for all pseudo-registers, if possible.
- If there are not enough registers in the machine, choose registers to spill to memory

Example



II. An Abstraction for Allocation & Assignment

- **Intuitively**
 - Two pseudo-registers **interfere** if at some point in the program they cannot both occupy the same register.
- **Interference graph**: an undirected graph, where
 - nodes = pseudo-registers
 - there is an edge between two nodes if their corresponding pseudo-registers interfere
- **What is not represented**
 - Extent of the interference between uses of different variables
 - Where in the program is the interference

Register Allocation and Coloring

- A graph is **n-colorable** if:
 - every node in the graph can be colored with one of the n colors such that two adjacent nodes do not have the same color.
- **Assigning n register (without spilling) = Coloring with n colors**
 - assign a node to a register (color) such that no two adjacent nodes are assigned same registers(colors)
- **Is spilling necessary? = Is the graph n-colorable?**
- **To determine if a graph is n-colorable is NP-complete, for $n > 2$**
 - Too expensive
 - Heuristics

III. Algorithm

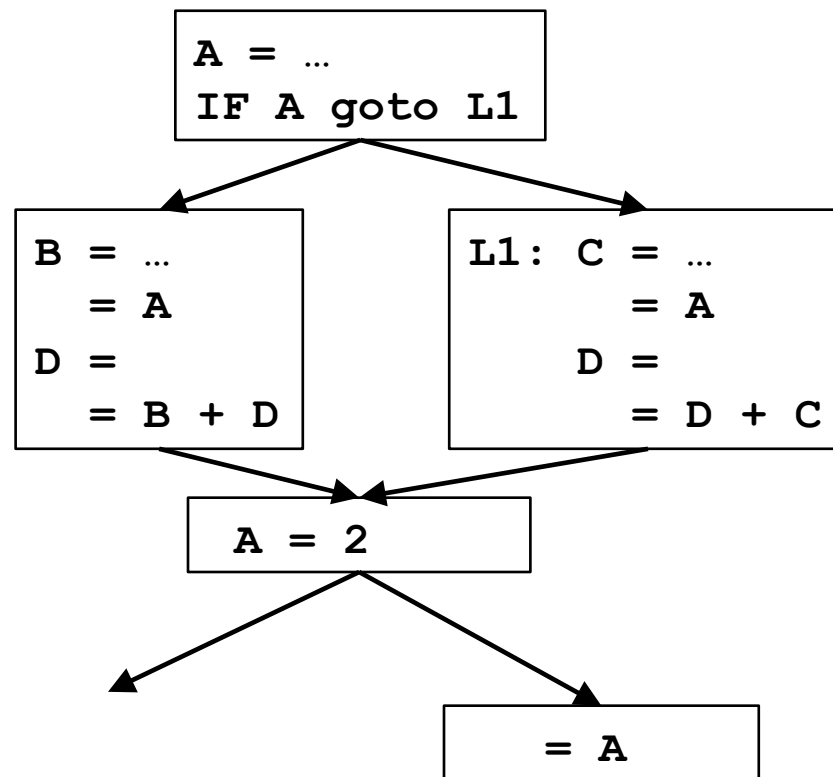
Step 1. Build an interference graph

- a. refining notion of a node
- b. finding the edges

Step 2. Coloring

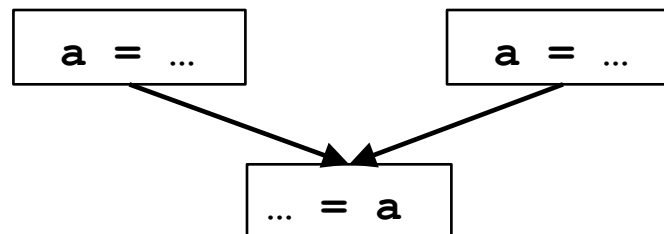
- use heuristics to try to find an n-coloring
 - Success:
 - colorable and we have an assignment
 - Failure:
 - graph not colorable, or
 - graph is colorable, but it is too expensive to color

Step 1a. Nodes in an Interference Graph

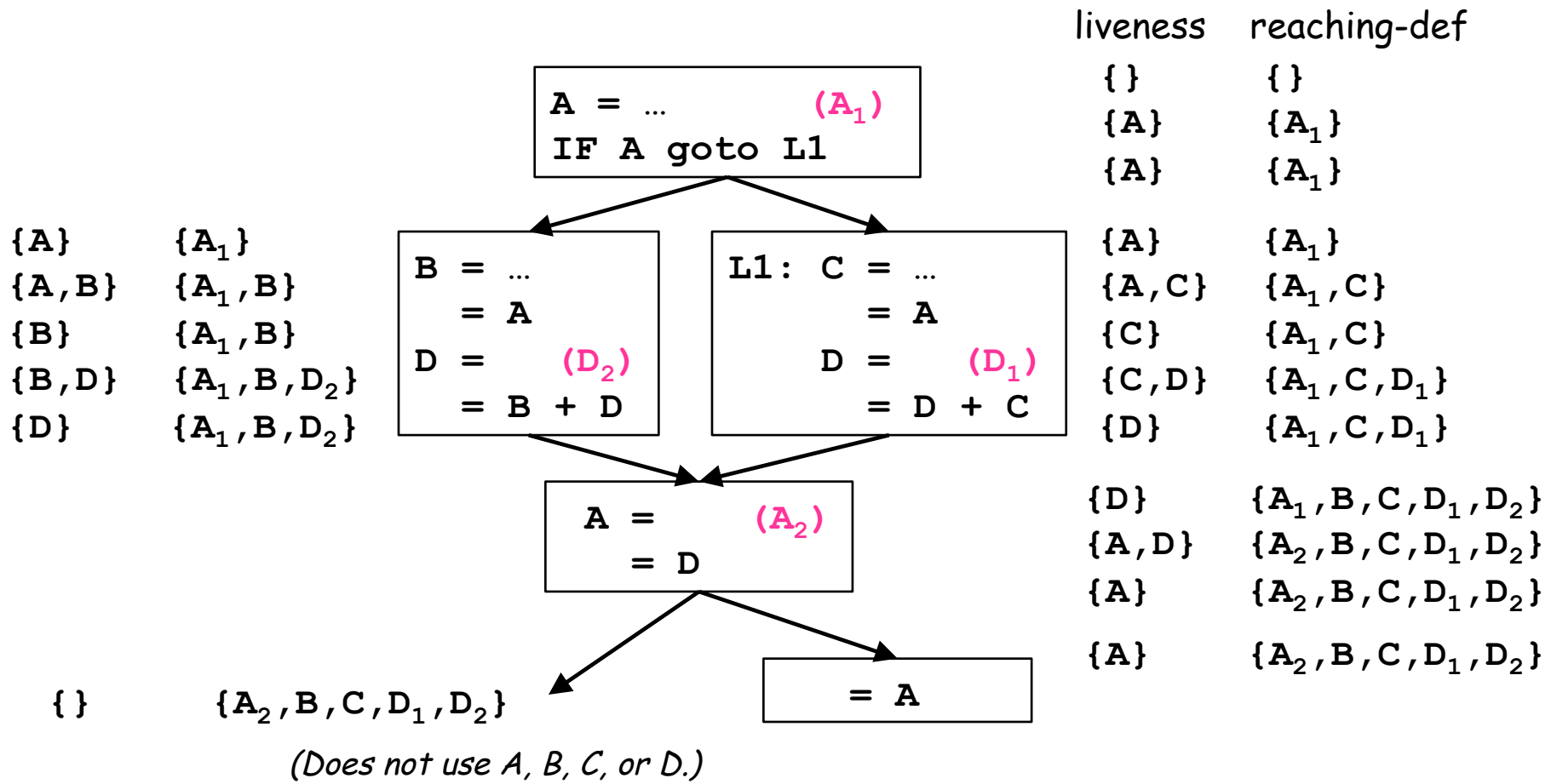


Live Ranges and Merged Live Ranges

- **Motivation:** to create an interference graph that is easier to color
 - Eliminate interference in a variable's "dead" zones.
 - Increase flexibility in allocation:
 - can allocate same variable to different registers
- A **live range** consists of a definition and all the points in a program (e.g. end of an instruction) in which that definition is live.
 - How to compute a live range?
- Two overlapping live ranges for the **same** variable must be merged



Example (Revisited)



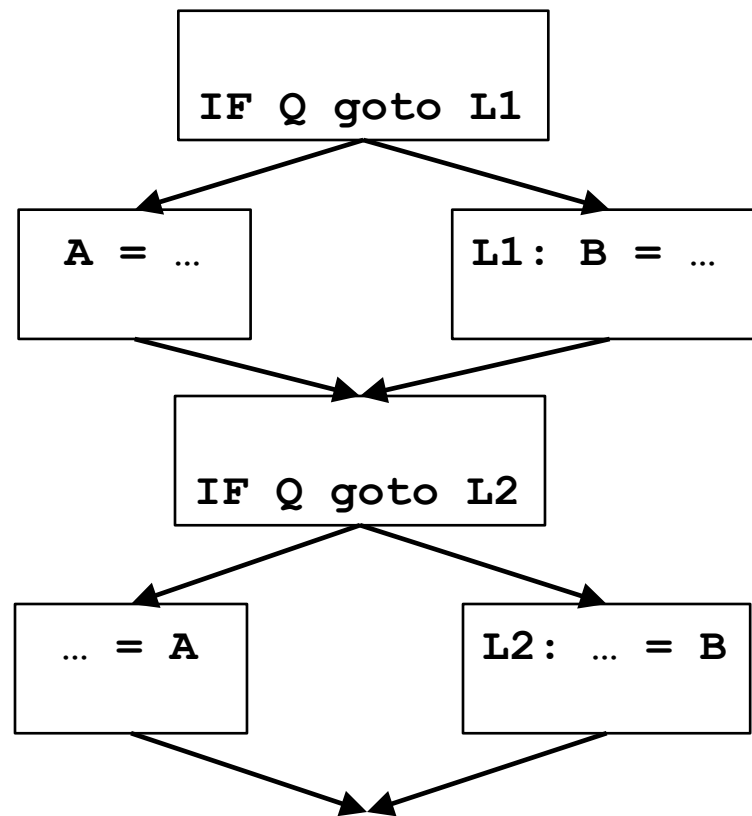
Merging Live Ranges

- **Merging definitions into equivalence classes**
 - Start by putting each definition in a different equivalence class
 - For each point in a program:
 - if (i) variable is live, and (ii) there are multiple reaching definitions for the variable, then:
 - merge the equivalence classes of all such definitions into one equivalence class
- **From now on, refer to merged live ranges simply as live ranges**

Step 1b. Edges of Interference Graph

- **Intuitively:**
 - Two live ranges (necessarily of different variables) may interfere if they overlap at some point in the program.
 - Algorithm:
 - At each point in the program:
 - enter an edge for every pair of live ranges at that point.
- **An optimized definition & algorithm for edges:**
 - Algorithm:
 - check for interference only at the start of each live range
 - Faster
 - Better quality

Example 2

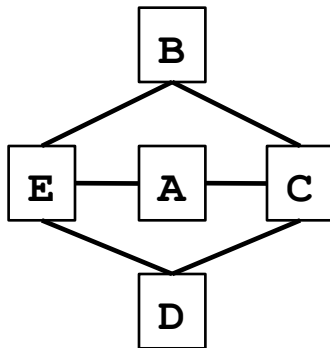


Step 2. Coloring

- **Reminder: coloring for $n > 2$ is NP-complete**
- **Observations:**
 - a node with degree $< n \Rightarrow$
 - can always color it successfully, given its neighbors' colors
 - a node with degree $= n \Rightarrow$
 - a node with degree $> n \Rightarrow$

Coloring Algorithm

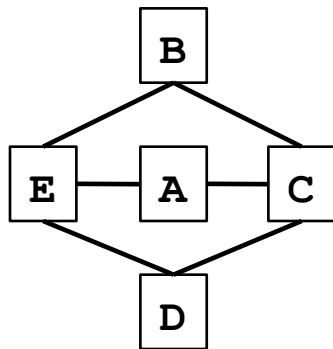
- **Algorithm:**
 - Iterate until stuck or done
 - Pick any node with degree $< n$
 - Remove the node and its edges from the graph
 - If done (no nodes left)
 - reverse process and add colors
- **Example ($n = 3$):**



- **Note: degree of a node may drop in iteration**
- **Avoids making arbitrary decisions that make coloring fail**

What Does Coloring Accomplish?

- **Done:**
 - colorable, also obtained an assignment
- **Stuck:**
 - colorable or not?



What if Coloring Fails?

- Use heuristics to improve its chance of success and to spill code

Build interference graph

Iterative until there are no nodes left

If there exists a node v with less than n neighbor
place v on stack to register allocate

else

v = node chosen by heuristics

(least frequently executed, has many neighbors)

place v on stack to register allocate (mark as spilled)

remove v and its edges from graph

While stack is not empty

Remove v from stack

Reinsert v and its edges into the graph

Assign v a color that differs from all its neighbors

(guaranteed to be possible for nodes not marked as spilled)

Summary

- **Problems:**
 - Given n registers in a machine, is spilling avoided?
 - Find an assignment for all pseudo-registers, whenever possible.
- **Solution:**
 - **Abstraction:** an **interference graph**
 - nodes: **live ranges**
 - edges: presence of live range at time of definition
 - **Register Allocation and Assignment** problems
 - equivalent to **n -colorability** of interference graph
 - **NP-complete**
 - **Heuristics** to find an assignment for n colors
 - successful: colorable, and finds assignment
 - not successful: colorability unknown & no assignment