Lecture 6

Register Allocation

I. Introduction
II. Abstraction and the Problem
III. Algorithm

Reading: Chapter 8.8.4
Before next class: Chapter 10.1 - 10.2

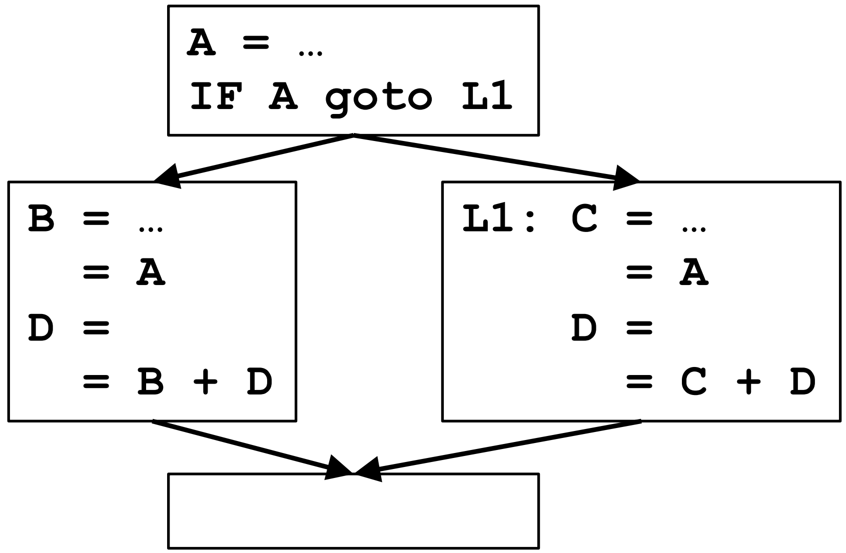
I. Motivation

- **Problem**
  - Allocation of variables (pseudo-registers) to hardware registers in a procedure
- **Perhaps the most important optimization**
  - Directly reduces running time
    - memory access ➔ register access
  - Useful for other optimizations
    - e.g. PRE assumes old values are kept in registers
- **General Lessons**
  - How to abstract a problem according to program/machine characteristics?
  - important & often-overlooked approach to NP-Complete problems

Goal

- **Find an assignment for all pseudo-registers, if possible.**
  - Not trying to minimize the number of registers used
- **If there are not enough registers in the machine, choose registers to spill to memory**

Example



II. An Abstraction for Allocation & Assignment

- **Intuitively**
  - Two pseudo-registers **interfere** if at some point in the program they cannot both occupy the same register.

- **Interference graph**: an undirected graph, where
  - nodes = pseudo-registers
  - there is an edge between two nodes if their corresponding pseudo-registers interfere

- **What is not represented**
  - The extent of the interference between uses of different variables
  - Where in the program is the interference

Quiz: Why is this a good representation?

Register Allocation and Coloring

- A graph is ***n*-colorable** if:
  - every node in the graph can be colored with one of the *n* colors such that two adjacent nodes do not have the same color.

- **Assigning n register (without spilling) = Coloring with n colors**
  - assign a node to a register (color) such that no two adjacent nodes are assigned same registers(colors)

- **Is spilling necessary? = Is the graph n-colorable?**

- **To determine if a graph is n-colorable is NP-complete, for n>2**
    - Too expensive
    - Heuristics

Quick Notes on NP-Completeness

- NP = P?
  - P: Polynomial
  - NP: Non-deterministic Polynomial
    - Exponential time on deterministic machines
  - A famous open problem in theory (unsolved after much research)

- NP-complete problems
  - If any can be solved in polynomial time, then NP = P

- Proving a problem is NP-complete → License to use heuristics

III. Algorithm

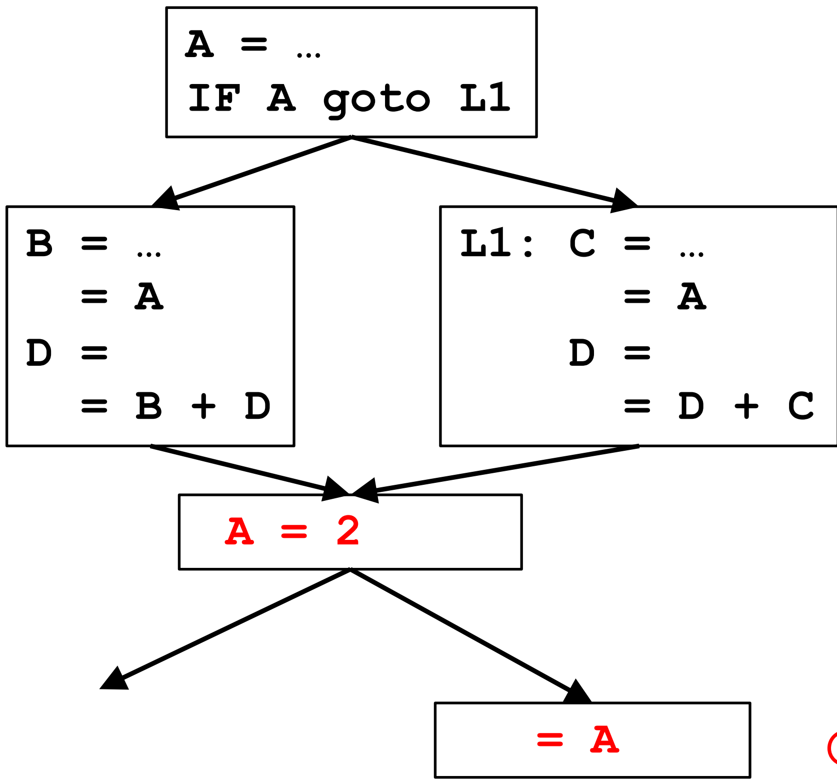
Step 1. Build an interference graph

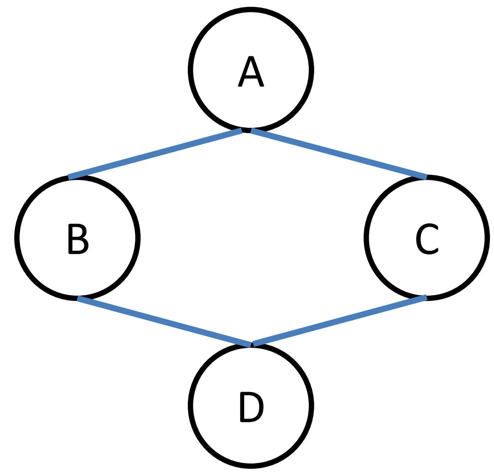
a. refining notion of a node

b. finding the edges

Step 2. Coloring Algorithm (NP-Complete Problem)

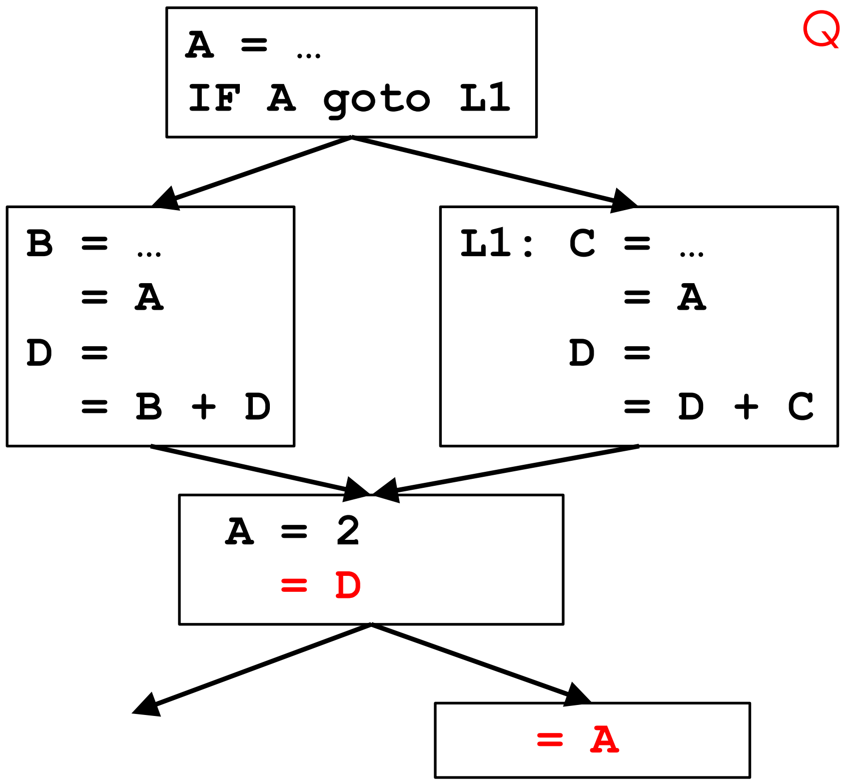
Step 1a. Nodes in an Interference Graph



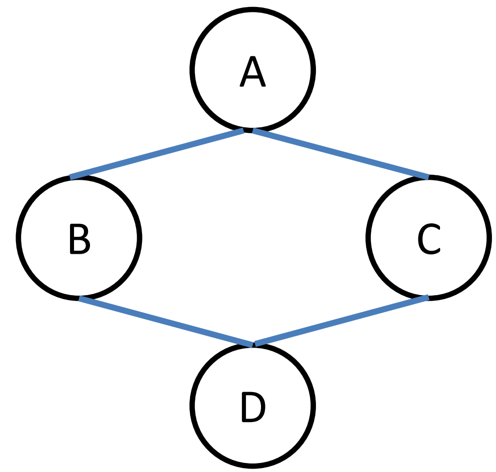


Quiz: What is the new inference graph with the additional code?

Step 1a. Nodes in an Interference Graph



Quiz: What is the new inference graph with the additional code?

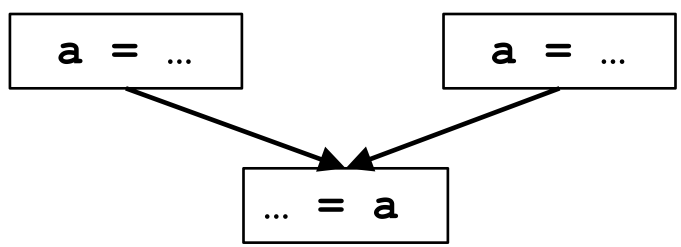


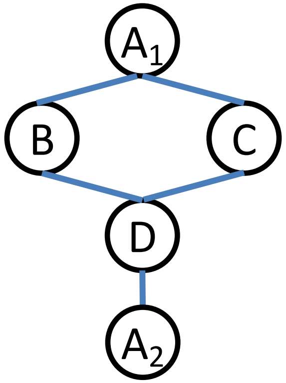
Live Ranges and Merged Live Ranges

- **Motivation: to create an interference graph that is easier to color**
  - Eliminate interference in a variable's "dead" zones.
  - Increase flexibility in allocation:
    - can allocate same variable to different registers

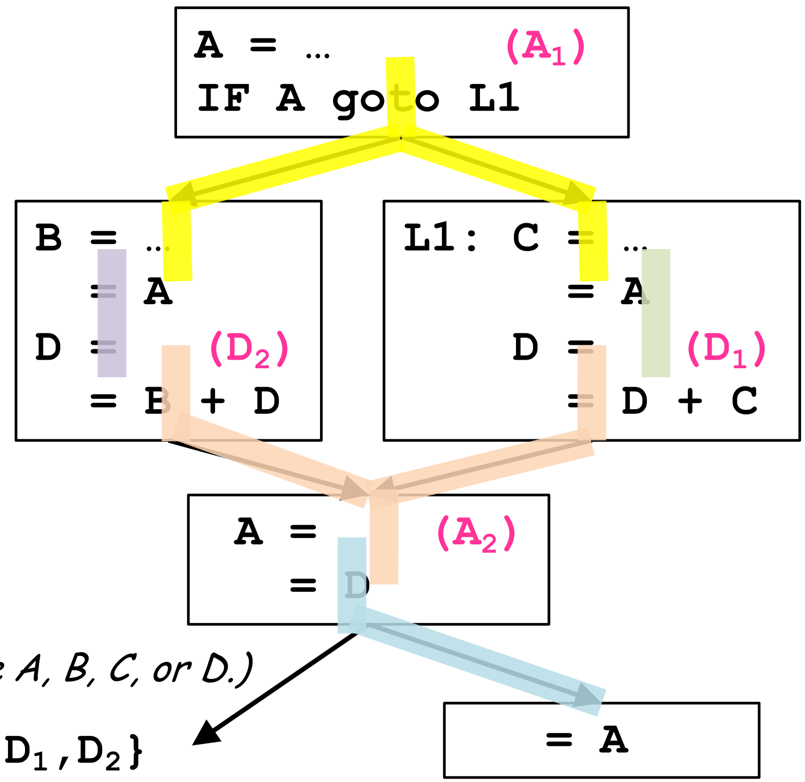
- A **live range** consists of a definition and all the points in a program (e.g. end of an instruction) in which that definition is live.
  - How to compute a live range? (homework)

- Two overlapping live ranges for the **same** variable must be merged





Example (Revisited)



(Does not use A, B, C, or D.)

{A}	$\{A_1\}$
{A,B}	$\{A_1,B\}$
{B}	$\{A_1,B\}$
{B,D}	$\{A_1,B,D_2\}$
{D}	$\{A_1,B,D_2\}$
{ }	$\{A_2,B,C,D_1,D_2\}$

liveness	reaching-def
{ }	{ }
{A}	$\{A_1\}$
{A}	$\{A_1\}$
{A}	$\{A_1\}$
{A,C}	$\{A_1,C\}$
{C}	$\{A_1,C\}$
{C,D}	$\{A_1,C,D_1\}$
{D}	$\{A_1,C,D_1\}$
{D}	$\{A_1,B,C,D_1,D_2\}$
{A,D}	$\{A_2,B,C,D_1,D_2\}$
{A}	$\{A_2,B,C,D_1,D_2\}$
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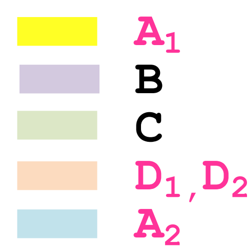


Merging Live Ranges

- **Merging definitions into equivalence classes**
  - Start by putting each definition in a different equivalence class
  - For each point in a program:
    - if (i) variable is live, and (ii) there are multiple reaching definitions for the variable, then:
      - merge the equivalence classes of all such definitions into one equivalence class

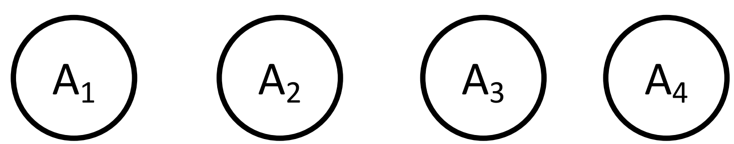
- **From now on, refer to merged live ranges simply as live ranges**

Given:

A_1 overlaps with A_2

A_3 overlaps with A_4

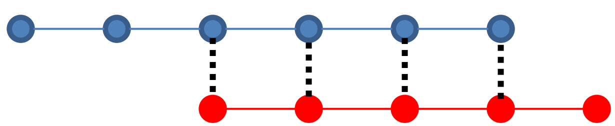
A_1 overlaps with A_3



Quiz: How many merged live ranges are here?

Step 1b. Edges of Interference Graph

- **Intuitively:**
  - Two live ranges (necessarily of different variables) may interfere if they overlap at some point in the program.
  - Algorithm:
    - At each point in the program:
      - enter an edge for every pair of live ranges at that point.



Live range 1

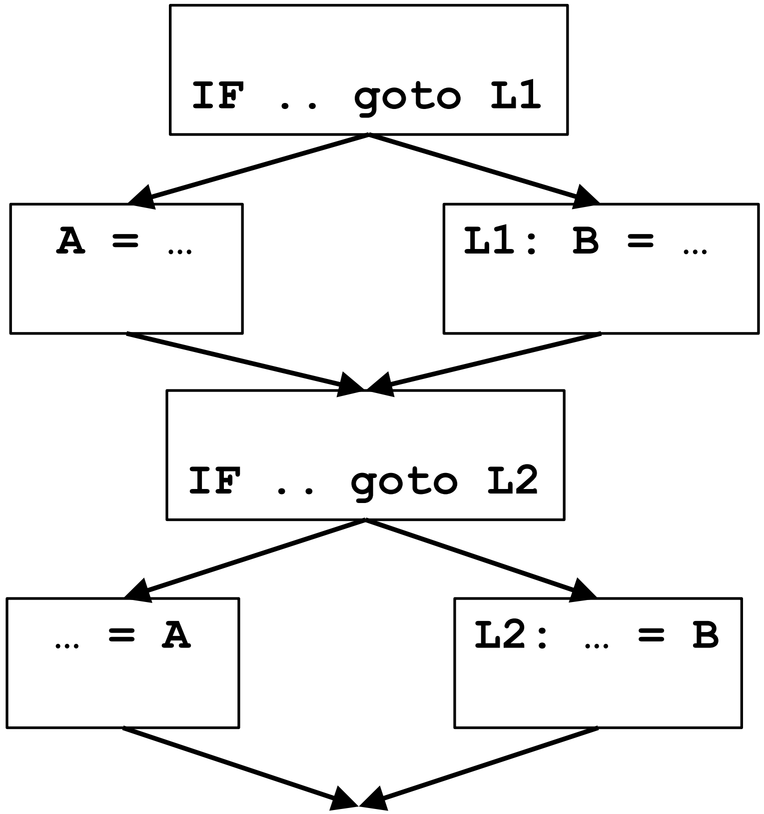
Live range 2

- **An optimized definition & algorithm for edges:**
  - Algorithm:
    - check for interference only at the starts of each merged live range
  - Faster
  - Better quality

Quiz: Is this correct?

Example 2

Quiz: How many registers do we need for A and B?



Lesson: Watch out for corner cases! Make sure the algorithm is correct!

Algorithm

Step 1. Build an interference graph

a. refining notion of a node

b. finding the edges

Step 2. Coloring Algorithm (NP-Complete Problem)

Quiz: What would you do?

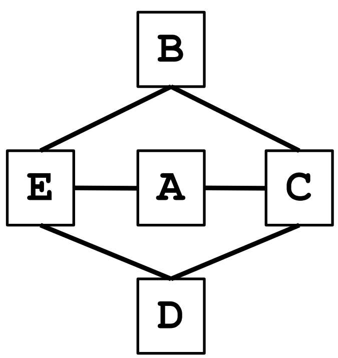
Observations

- **Reminder: coloring for n > 2 is NP-complete**

- **<u>Observations</u>:**
  - a node with degree < n ⇒
    - can always color it successfully, given its neighbors' colors
  - a node with degree = n ⇒
  - a node with degree > n ⇒

Coloring Algorithm

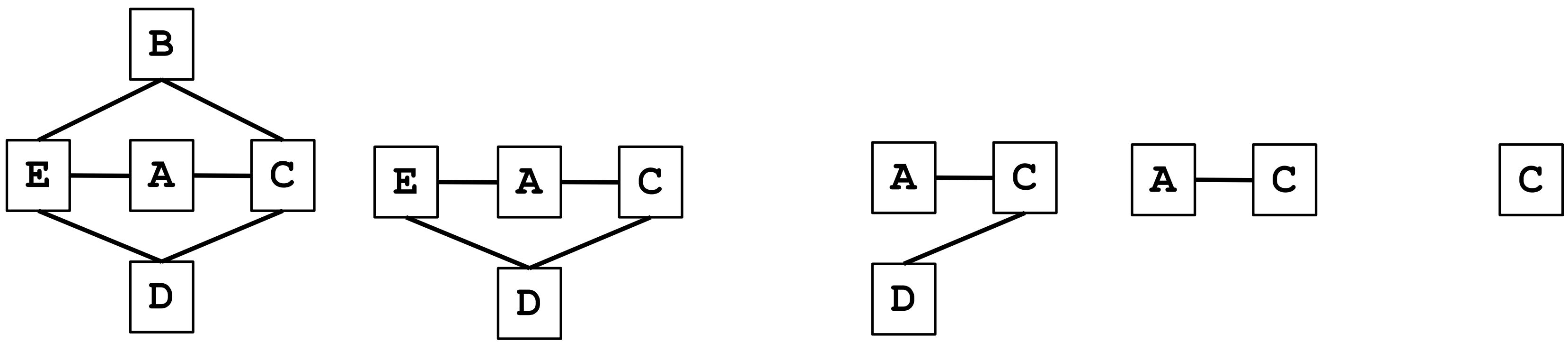
- **Algorithm:**
  - Iterate until stuck or done
    - Pick any node with degree < n
    - Remove the node and its edges from the graph
  - If done (no nodes left)
    - reverse process and add colors
- **Example (n = 3):**



- **Note: degree of a node may drop in iteration**
- **Avoids making arbitrary decisions that make coloring fail**

What Does Coloring Accomplish?

- **Done:**
  - colorable, also obtained an assignment
- **Stuck:**
  - colorable or not?
- Example: n = 2



Even if the algorithm gets stuck, it does not mean that it is not colorable.

What if Coloring Fails?

- **Use heuristics to improve its chance of success and to spill code**

Build interference graph

Iterative until there are no nodes left
    If there exists a node v with less than n neighbor
        place v on stack to register allocate
    else
        v = node chosen by heuristics
          (least frequently executed)
        place v on stack to register allocate (mark as spilled)
    remove v and its edges from graph

While stack is not empty
    Remove v from stack
    Reinsert v and its edges into the graph
    Assign v a color that differs from all its neighbors if possible
    (guaranteed to be possible only for nodes not marked as spilled)

Summary

- **Problems:**
  - Given n registers in a machine, is spilling avoided?
  - Find an assignment for all pseudo-registers, whenever possible.
- **Solution:**
  - Abstraction: an **interference graph**
    - nodes: live ranges
    - edges: presence of live range at time of definition
  - Register Allocation and Assignment problems
    - equivalent to **n-colorability** of interference graph
      ➔ NP-complete
  - Heuristics to find an assignment for n colors
    - successful: colorable, and finds assignment
    - not successful: colorability unknown & no assignment
- **General lessons**:
  - Problem abstraction depends on program/machine characteristics
  - Minimize making arbitrary decisions for NP-complete problems
  - Careful about corner cases