Lecture 5

Partial Redundancy Elimination

I. Redundancy Optimizations
  - Global common subexpression elimination
  - Loop invariant code motion
  - Partial redundancy elimination

II. Lazy Code Motion Algorithm
  - Mathematical concept: a cut set
  - Basic technique (anticipation)
  - 3 more passes to refine algorithm

Reading: Chapter 9.5

Jens Knoop, Oliver Rüthing, Berhard Steffen, Lazy Code Motion, PLDI 1992. (Most Influential PLDI Paper Award, 2002)

Overview

- **Redundancy optimizations**
  - Global common subexpression elimination
  - Loop invariant code motion
  - Partial redundancy elimination (subsumes the above)
    - Originally formulated as 1 bi-directional analysis!

- **Partial redundancy: Lazy code motion algorithm**
  - Formulated as 4 separate uni-directional passes
    - backward, forward, forward, backward
  - Easier to analyze and better performance than 1 bi-directional analysis

- **Shows off the power and elegance of data flow**
  - Sees program as a graph
  - Uses the math concept of a cut set, used later in instruction scheduling

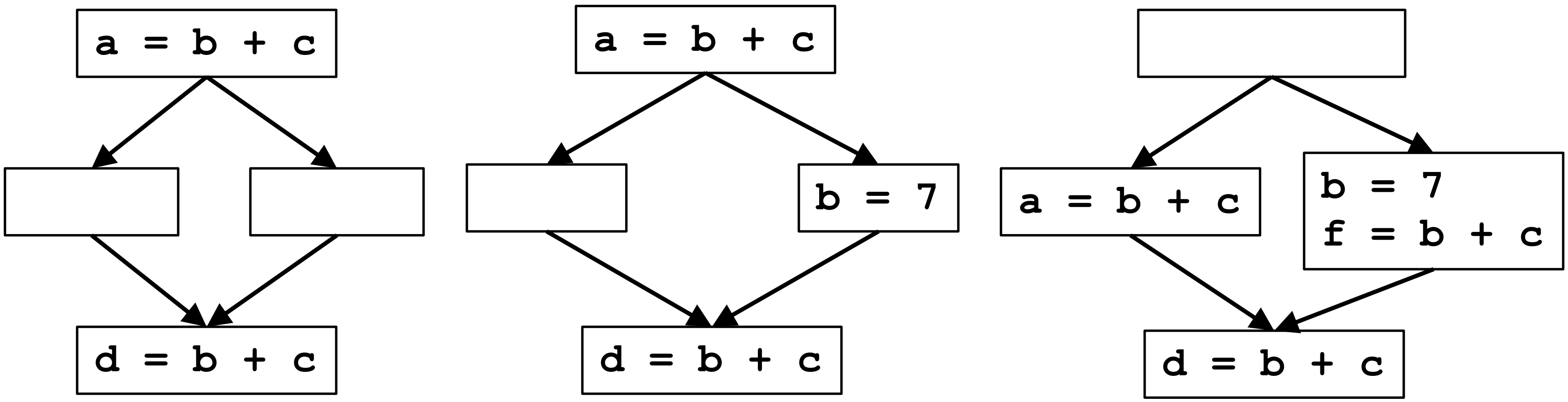
Outline of This Lecture

- **Overview**
  - Simple examples to build up your intuition
  - Introduce cut sets
  - Key: understand what the algorithm does without simulation
  - Details of the algorithm

- **Simple but hard**: Please work out examples after class immediately

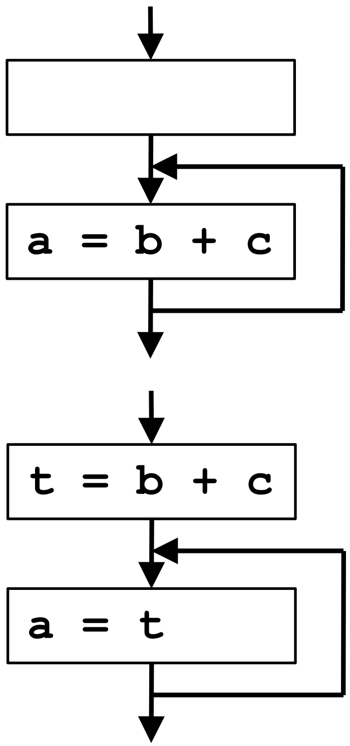
I. Common Subexpression Elimination

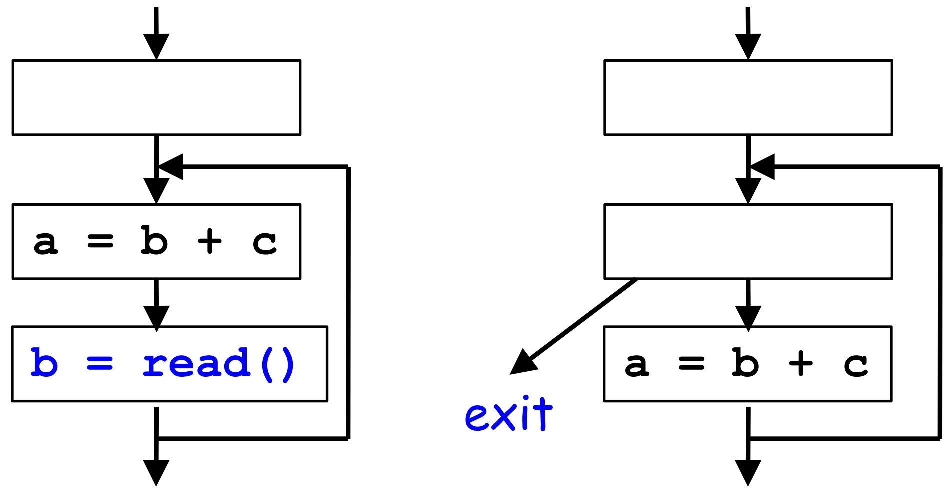
Build up intuition about redundancy elimination with examples of familiar concepts



- **A common expression may have different values on different paths!**
- **On every path reaching p,**
  - expression b+c has been computed
  - b, c not overwritten after the expression

Loop Invariant Code Motion



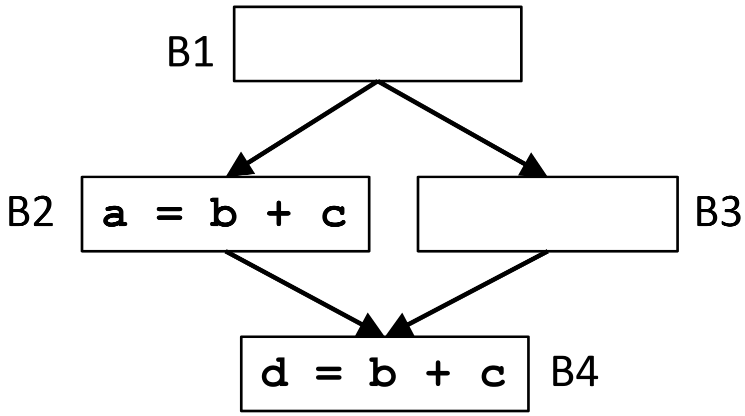


- Given an expression (b+c) inside a loop,
  - does the value of b+c change inside the loop?
  - is the code executed at least once?

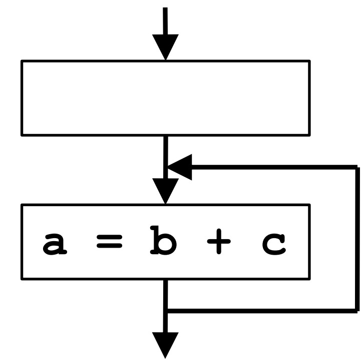
Observations:

- Important optimization. Why?
- Unlike common subexpression elimination,
  need to place an instruction in a new program point!
  Where?

Partial Redundancy



Observation:
Multiple possible placements of instructions. Where?

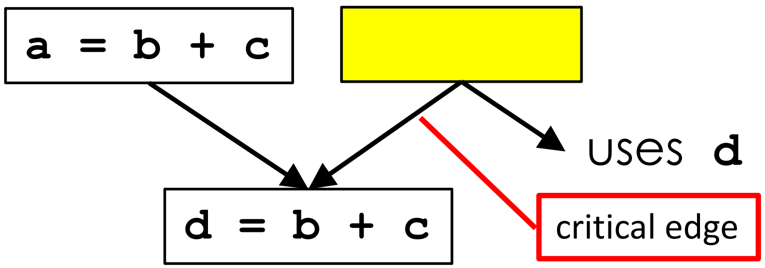


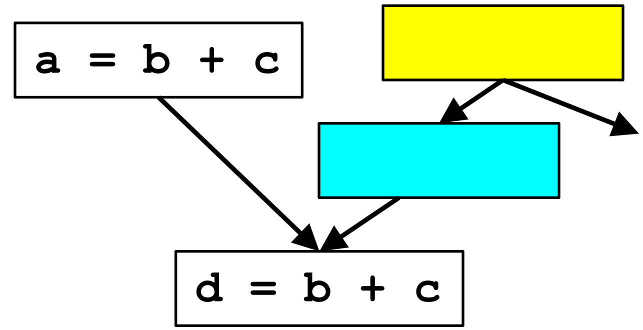
- Can we place calculations of b+c such that no path re-executes the same expression?
- Partial Redundancy Elimination (PRE)
  - subsumes:
    - global common subexpression (full redundancy)
    - loop invariant code motion (partial redundancy for loops)

Quiz: Why?

Unifying theory: More powerful, elegant → but less direct.

II. Preparing the Flow Graph

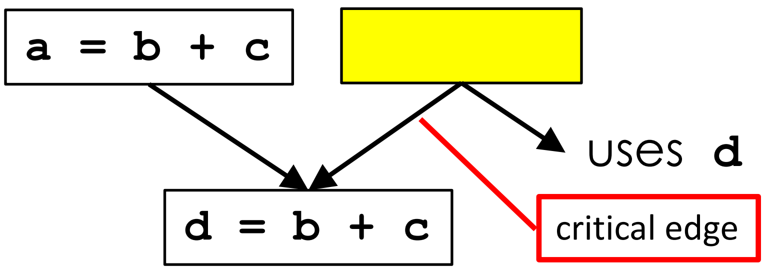


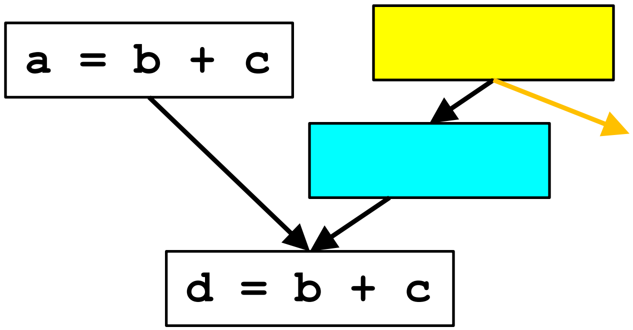


- Original PRE: a bi-direction data flow problem! Hard to understand
- **Problem: Critical edges**
  - Source basic block has multiple successors
  - Destination basic block has multiple predecessors

- **Lazy Code Motion (Knoop 92)** – most influential paper award
  - Suggest changing the graph: getting rid of critical edges
  - Increase opportunities for optimization
  - Replace the bi-directional data flow problem into 4 unidirectional data flows – simpler to understand.

II. Preparing the Flow Graph

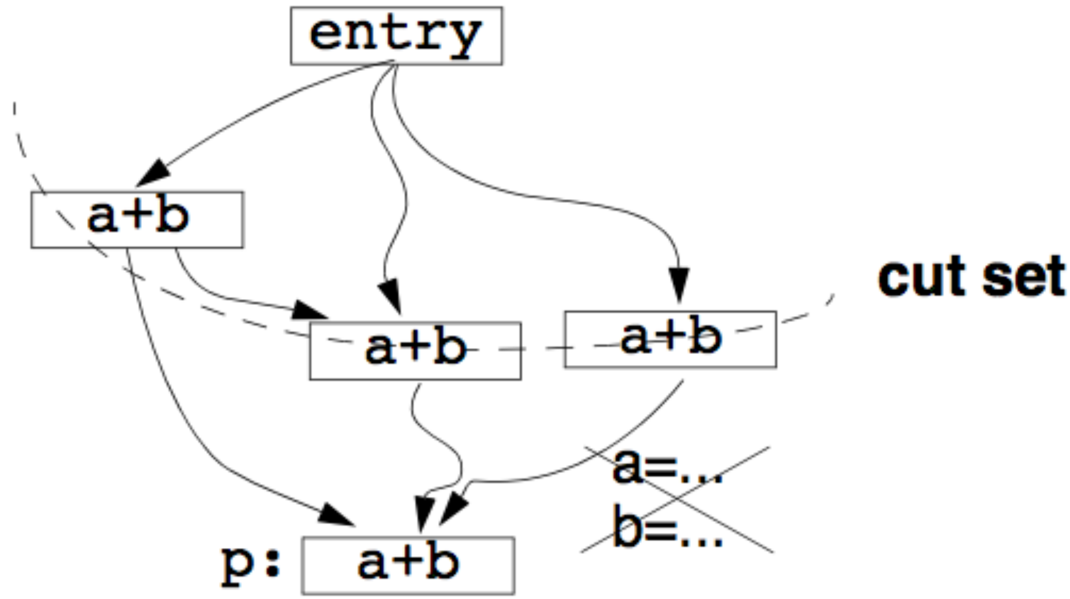




- **Definition: Critical edges**
  - source basic block has multiple successors
  - destination basic block has multiple predecessors
- **Modify the flow graph: (treat every statement as a basic block)**
  - To keep algorithm simple:
    restrict placement of instructions to the beginning of a basic block
  - Add a basic block for every edge that leads to a basic block with multiple predecessors (not just on critical edges)

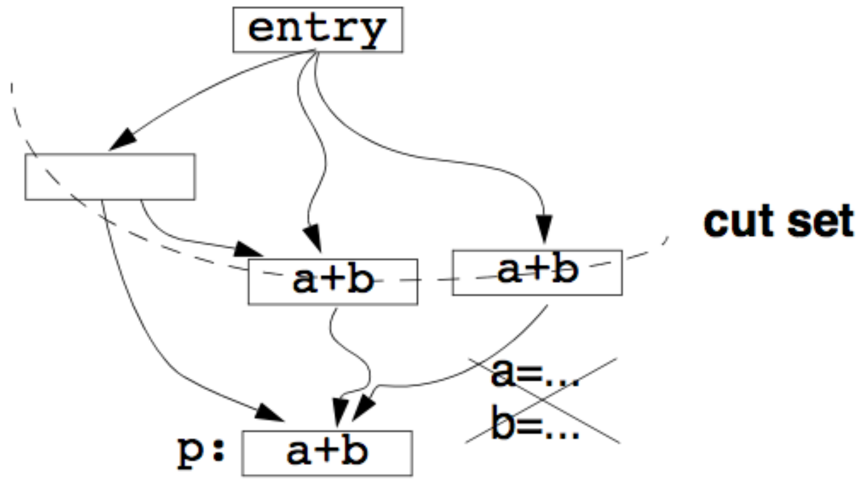
Full Redundancy: A Cut Set in a Graph

Key mathematical concept



- **A cut set of node *p*: a set of nodes that separate the start node from *p***
- **Full redundancy at *p*: expression a+b redundant on all paths**
  - there exists a cut set containing calculations of a+b
  - and a, b, are not redefined

Partial Redundancy: Completing a Cut Set



- **Partial redundancy at p: redundant on some but not all paths**
  - Add operations to create a cut set containing a+b
- **Constraint on placement: no wasted operation**
  - a+b is anticipated at B if its value computed at B will be used along ALL subsequent paths
  - a, b not redefined before use, no branches that lead to exit without using a+b
- **Range where a+b is anticipated → Choice**
- **Greedy**: Place operations at the earliest frontier to minimize redundancy

Pass 1: Anticipated Expressions

This pass does most of the heavy lifting in eliminating redundancy

- **Backward pass: Anticipated expressions**
  Anticipated[b].in: Set of expressions anticipated at the entry of b
    - An expression is anticipated if its value computed at point p will be used along ALL subsequent paths

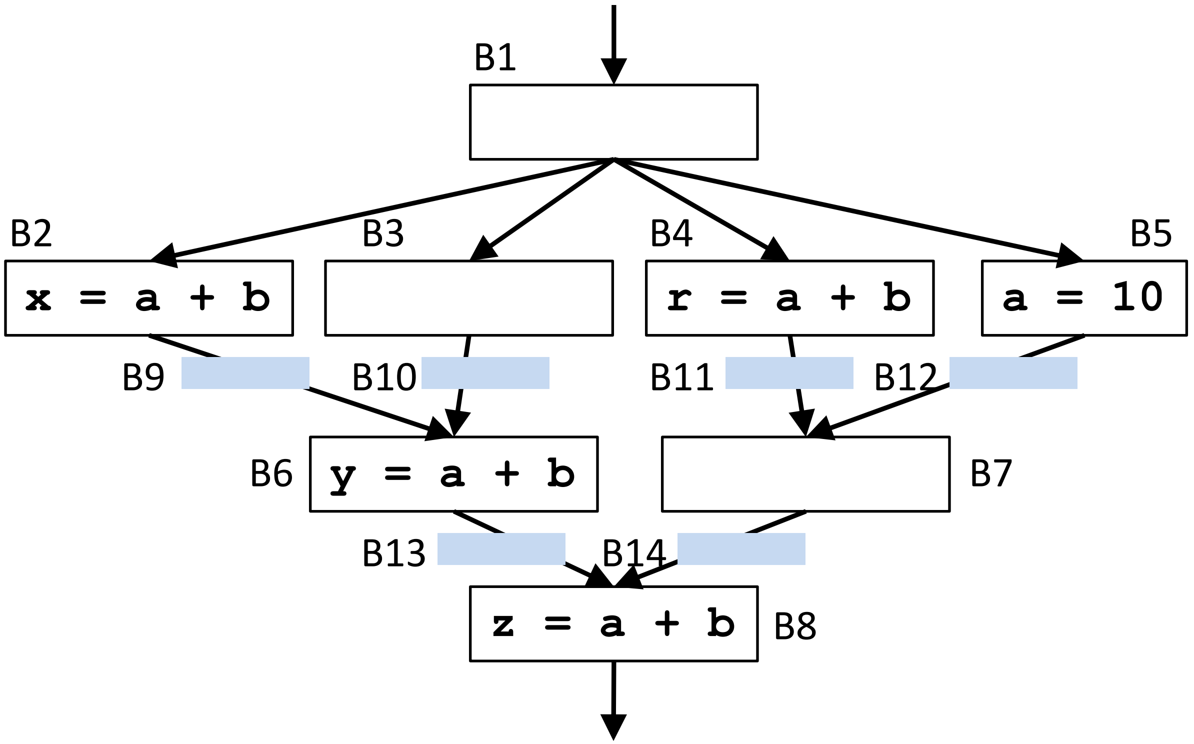
	Anticipated Expressions
Domain	Sets of expressions
Direction	backward
Transfer Function	$f_b(x) = EUse_b \cup (x - EKill_b)$ EUse: used exp, EKill: exp killed
$\wedge$	$\cap$
Boundary	in[exit] = $\emptyset$
Initialization	in[b] = {all expressions}

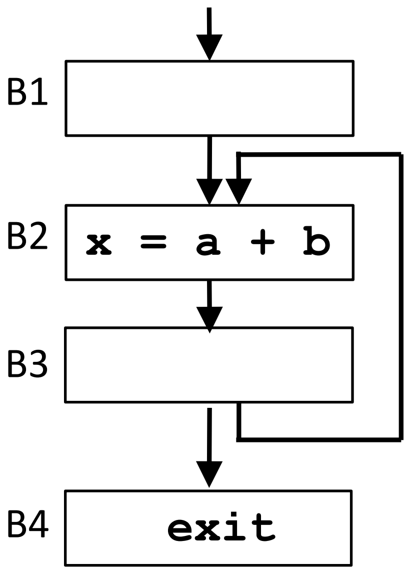
e.g. c=a+b
EUse: a+b
EKill: all exp using c

- **First approximation:**
    - place operations at the earliest frontier of anticipation (boundary from “not anticipated” to “anticipated”)

Examples (1)

See the algorithm in action

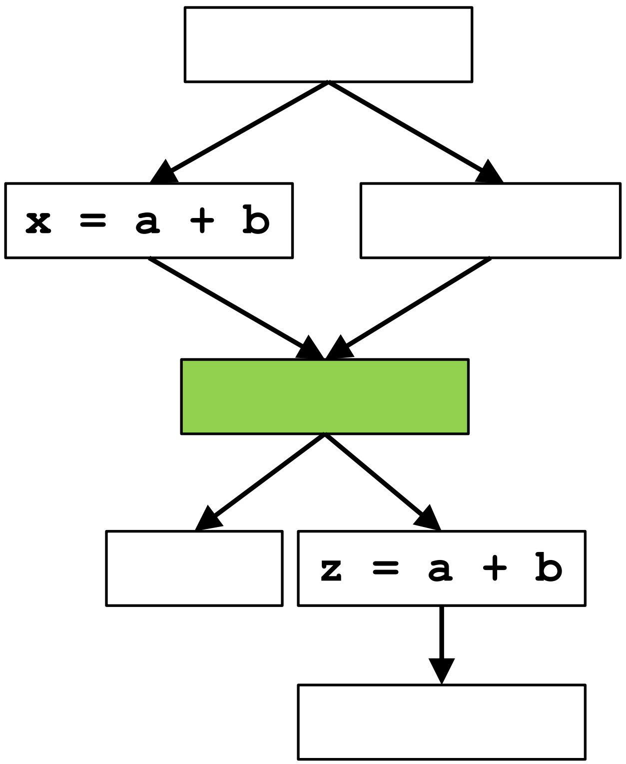




(Added basic blocks assumed; not shown for simplicity)

Examples (2)

Can Partial Redundancy Elimination eliminate all partial redundancy?

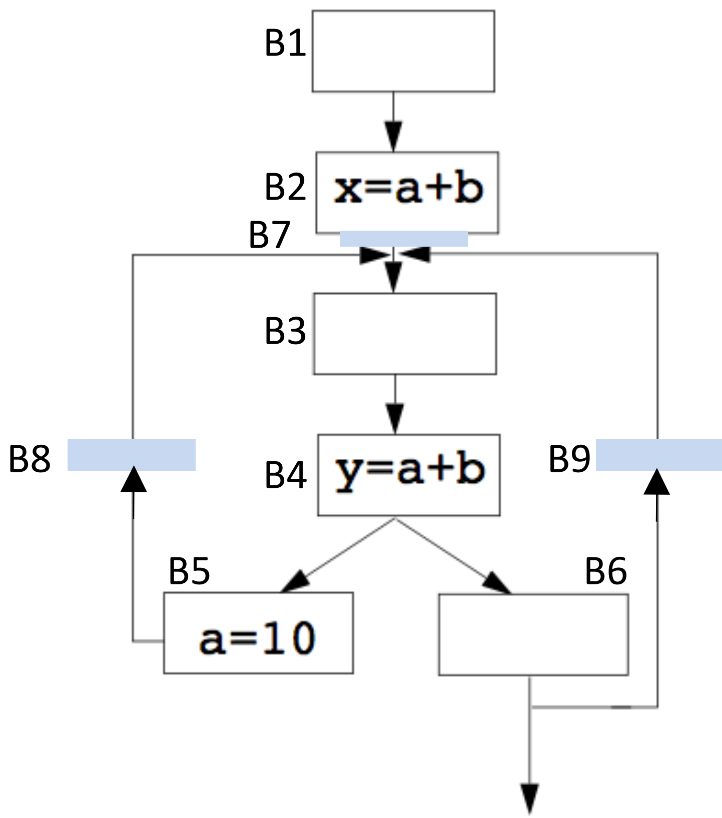


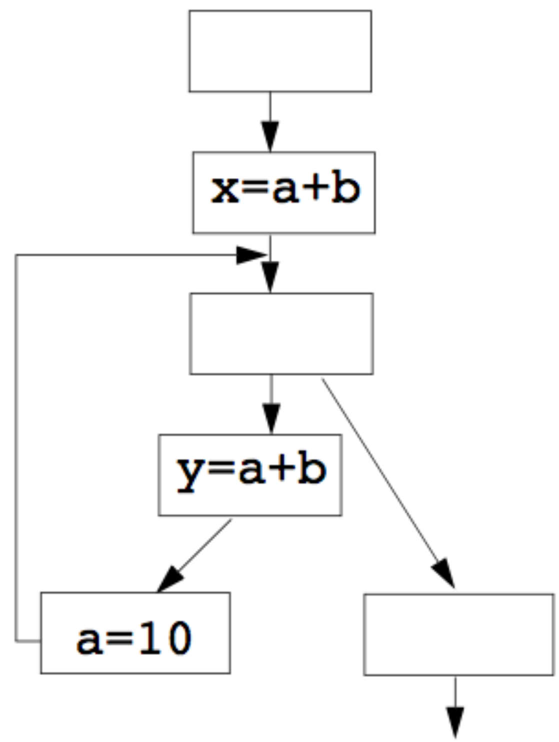
Code Duplication

- Code duplication can reduce redundancy
- Cost: larger code size
  → can be costly if it causes more misses in the instruction cache
- Code duplication not used for partial redundancy elimination
  - Used in instruction scheduling for parallelism

Examples (3)

Do you know how the algorithm works without simulating it?

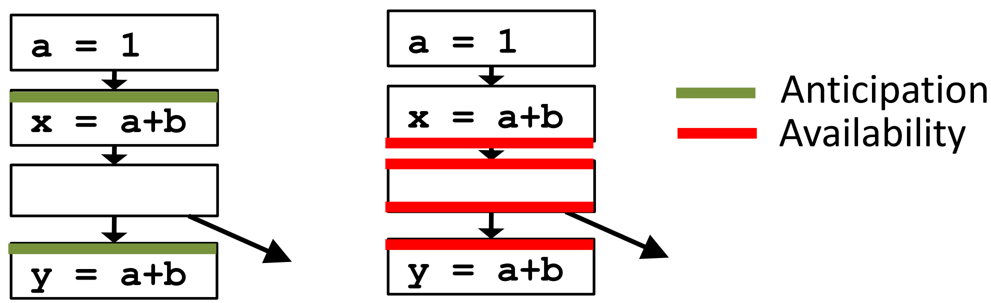




Pass 2: Place As Early As Possible

There is still some redundancy left!

- **First approximation: frontier between "not anticipated" & "anticipated"**
- **Complication: Anticipation may oscillate**



- **An anticipation frontier may cover a subsequent frontier.**
  - Once an expression has been anticipated (and assumed evaluated) it is "available" to subsequent frontiers
    → no need to re-evaluate.
- e will be **available at p** if
  e has been "anticipated but not subsequently killed" on all paths reaching p
- **Place expression at the earliest point anticipated and not already available**
  - earliest(b) = anticipated[b].in - available[b].in

Available Expressions

- e will be **available at p** if
  e has been "anticipated but not subsequently killed" on all paths reaching p

	Available Expressions
Domain	Sets of expressions
Direction	forward
Transfer Function	$f_b(x) = (\text{Anticipated}[b].\text{in} \cup x) - \text{EKill}_b$
$\wedge$	$\cap$
Boundary	out[entry] = $\varnothing$
Initialization	out[b] = {all expressions}

Early Placement

- **earliest(b)**
  - set of expressions added to block b under early placement
- **Place expression at the**
  earliest point anticipated and not already available
  - earliest(b) = anticipated[b].in - available[b].in
- **Algorithm**
  - For all basic block b,

    if x+y ∈ earliest[b]

      at beginning of b:

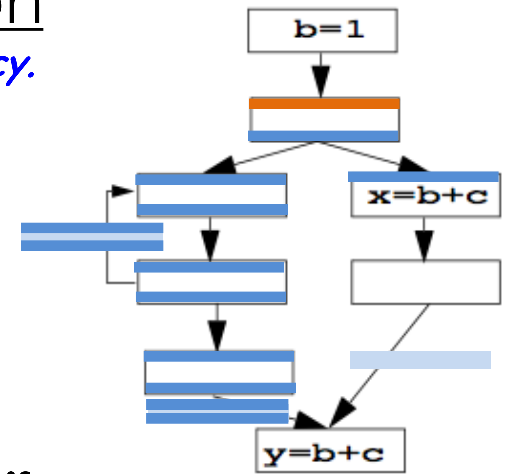
        let t be the unique variable representing x+y
        add t = x+y,

    replace every original x+y in the program by t

Pass 3: Lazy Code Motion

Let's be lazy without introducing redundancy.

Delay without creating redundancy to reduce register pressure



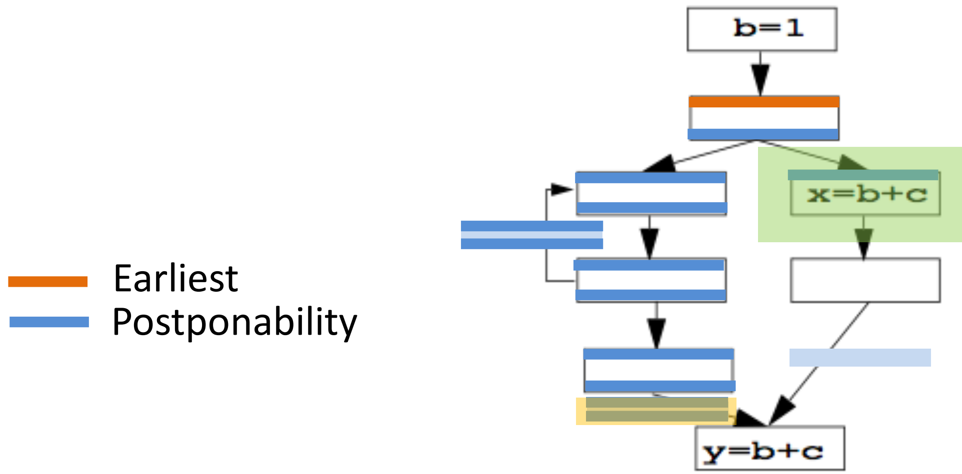
Earliest
Postponability

An expression e is postponable at a program point p if

- all paths leading to p
  have seen the earliest placement of e but not a subsequent use

	Postponable Expressions
Domain	Sets of expressions
Direction	forward
Transfer Function	$f_b(x) = (earliest[b] \cup x) - EUse_b$
$\wedge$	$\cap$
Boundary	$out[entry] = \emptyset$
Initialization	$out[b] = \{all\ expressions\}$

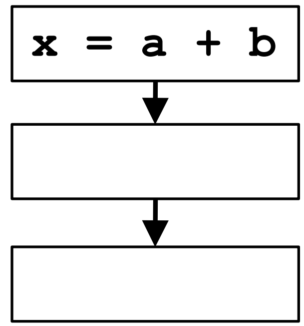
Latest: Latest Frontier of "postponable" Cut Set



- latest[b] = (earliest[b] ∪ postponable.in[b]) ∩
  (EUse$_b$ ∪ ¬($\bigcap_{s \in succ[b]}$(earliest[s] ∪ postponable.in[s])))
  - OK to place expression: earliest or postponable.in
  - Need to place at b if either
    - used in b, or
    - not OK to place in one of its successors
- Note: If postponable.out[b] and ¬ postponable.in[s],
  b is empty and there is only one successor s

Pass 4: Cleaning Up

Finally… this is easy, it is like liveness



not used afterwards

- Eliminate temporary variable assignments unused beyond the current block
- Compute: Used.out[b]: sets of used (live) expressions at exit of b.

	Used Expressions
Domain	Sets of expressions
Direction	backward
Transfer Function	$f_b(x) = (EUse[b] \cup x) - latest[b]$
$\wedge$	$\cup$
Boundary	$in[exit] = \emptyset$
Initialization	$in[b] = \emptyset$

Code Transformation

Original version: For each basic block b,

if x+y ∈ earliest[b]

at beginning of b:

let t be the unique variable representing x+y
add t = x+y,

replace every original x+y in the program by t

New version: For each basic block b,

if (x+y) ∈ (latest[b] ∩ ¬ used.out[b])){ }

else

if x+y ∈ latest[b]

at beginning of b:

let t be the unique variable representing x+y
add t = x+y,

replace every original x+y in the program by t

4 Passes for Partial Redundancy Elimination

- ***Heavy lifting*: Cannot introduce operations not executed originally**
  - Pass 1 (backward): **Anticipation**: range of code motion
  - Placing operations at the frontier of anticipation gets most of the redundancy
- ***Squeeze the last drop of redundancy*:**
  An anticipation frontier may cover a subsequent frontier
  - Pass 2 (forward): **Availability**
  - **Earliest**: anticipated, but not yet available
- ***Push the cut set out -- as late as possible***
  To minimize register lifetimes
  - Pass 3 (forward): **Postponability**: move it down provided it does not create redundancy
  - **Latest**: where it is used or the frontier of postponability
- ***Clean up*: Remove unused assignment**
  - Pass 4: **Used**: if not used, don't do anything

Remarks

- **Powerful algorithm**
  - Finds many forms of redundancy in one unified framework
- **Illustrates the power of data flow**
  - Multiple data flow problems