Lecture 4

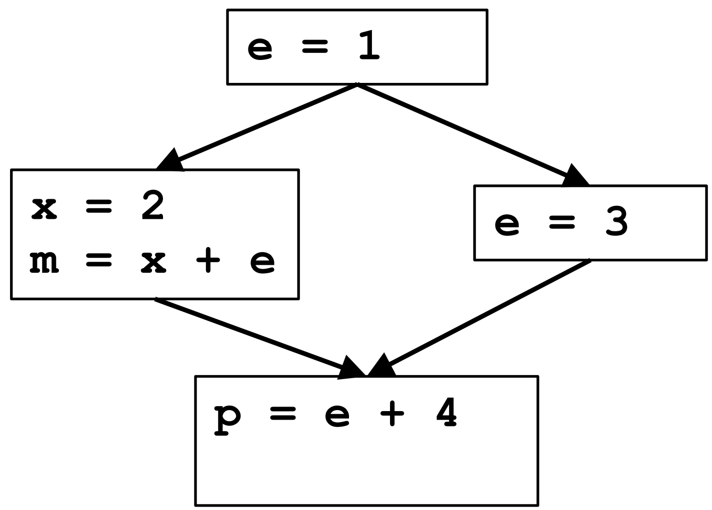
Advanced Topics:
Constant Propagation
Speed

I. Constant Propagation
II. Efficiency of Data Flow Analysis

Reading: Chapter 9.4

I. Constant Propagation/Folding

- **At every basic block boundary, for each variable v**
  - determine if v is a constant
  - if so, what is the value?



Semi-lattice Diagram

Example Flow Graphs

- Finite domain?
- Finite height?

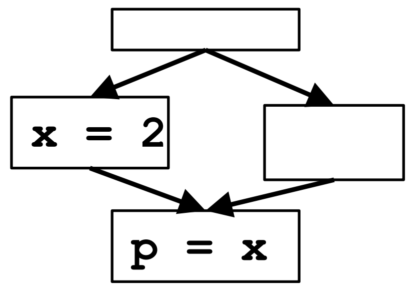
Equivalent Definition (with a Meet Operator)

- **Meet Operator:**

v1	**v2**	**v1 $\wedge$ v2**
undef	undef	
	c_2	
	NAC	
c_1	undef	
	c_2	
	NAC	
NAC	undef	
	c_2	
	NAC	

  – Note: undef $\wedge$ c2 = c2!

Example



Transfer Function

- **Assume a basic block has only 1 instruction**
- **Let IN[b,x], OUT[b,x]**
  - be the information for variable x at entry and exit of basic block b
- **OUT[entry, x] = undef, for all x.**
- **Non-assignment instructions: OUT[b,x] = IN[b,x]**
- **Assignment instructions: (next page)**

Constant Propagation (Cont.)

- **Let an assignment be of the form $x_3 = x_1 + x_2$**
  - "+" represents a generic operator
  - OUT[b,x] = IN [b,x], if $x \neq x_3$

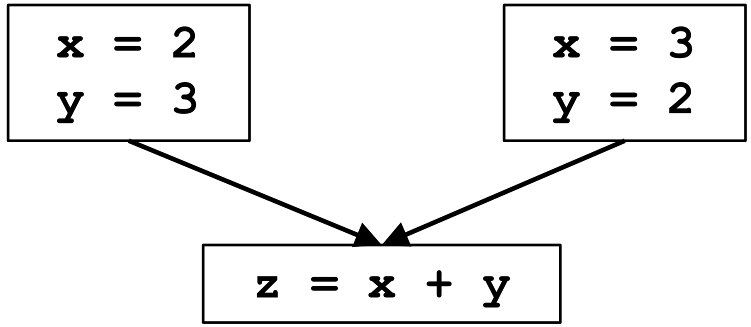
IN[b,x_1]	IN[b,x_2]	OUT[b,x_3]
undef	undef	
	c_2	
	NAC	
c_1	undef	
	c_2	
	NAC	
NAC	undef	
	c_2	
	NAC	

Is this monotone?

1. Hold x_1 constant, lower x_2, show results do not rise.
2. Hold x_2 constant, lower x_1, show results do not rise.
3. The combination proves monotonicity.

- **Use:** $x \leq y$ implies $f(x) \leq f(y)$ to check if framework is monotone
  - $[v_1\ v_2 \ldots] \leq [v_1'\ v_2' \ldots]$, $f([v_1\ v_2 \ldots]) \leq f([v_1'\ v_2' \ldots])$

Distributive?

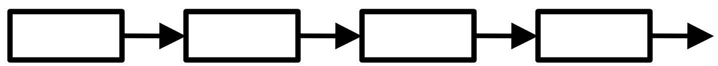


Summary of Constant Propagation

- **A useful optimization**
- **Illustrates:**
  - abstract execution
  - an infinite semi-lattice
  - a non-distributive problem

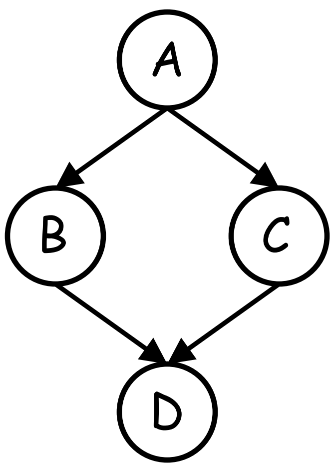
II. Efficiency of Iterative Data Flow

- **Assume forward data flow for this discussion**
- **Speed of convergence depends on the ordering of node**

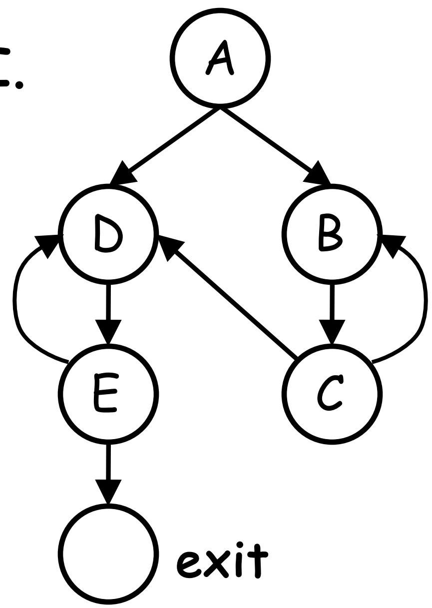


- **How about:**

I.



II.



Depth-first Ordering: Reverse Postorder

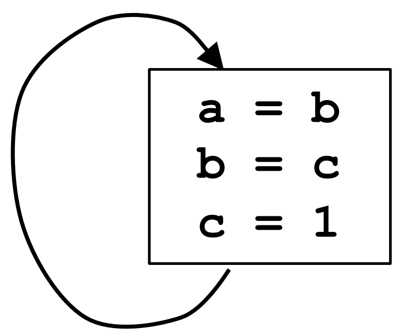
- **Preorder traversal: visit the parent before its children**
- **Postorder traversal: visit the children then the parent**
- **Preferred ordering: reverse postorder**
- **Intuitively**
  - depth first postorder visits the farthest node as early as possible
  - reverse postorder delays visiting farthest node

“Reverse Post-Order” Iterative Algorithm

```
input: control flow graph CFG = (N, E, Entry, Exit)

// Boundary condition
 OUT[Entry] = ∅

// Initialization for iterative algorithm
 For each basic block B other than Entry
    OUT[B] = ∅

// iterate
 While (changes to any OUT occur) {
   For each basic block B other than Entry in reverse post order {
      IN[B] = ∪ (OUT[p]), for all predecessors p of B
      OUT[B] = fB(IN[B])   // OUT[B]=gen[B]∪(IN[B]-kill[B])
  }
```

Consideration of Speed of Convergence

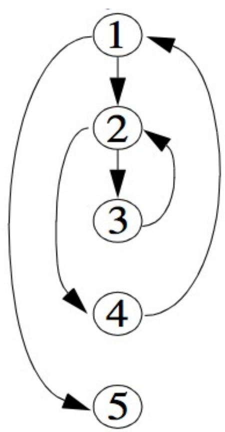
- **Does it matter if we go around the same cycle multiple times?**
- **Reachability problems: “Does a path exist?”**
  - Reaching definitions, liveness
  - Does not matter how many times we go around cycles
- **Traversing cycles can make a difference:** constant propagation



Speed of Convergence

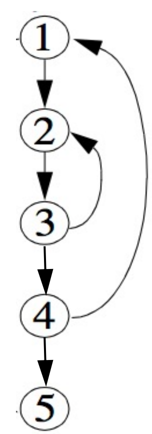
- If cycles do not add info:
  - Labeling nodes in a path by their reverse postorder rank:

    1 -> 4 -> 5 -> 7 -> 2 -> 4 ...
  - info flows down nodes of increasing reverse postorder rank in 1 pass
- Loop depth = max. # of “retreating edges” in any acyclic path
- **Maximum** # iterations in data flow algorithm = Loop depth+2
  (2 is necessary even if there are no cycles)



Loop
depth =

Iterations
needed =



Loop
depth =

Iterations
needed =

- May not need the maximum depth
- Knuth showed: average loop depth in real functions = 2.75

Summary

- **Constant propagation**
  - abstract execution
  - an infinite semi-lattice
  - a non-distributive framework
- **Convergence**
  - **Reverse postorder iterative algorithm**
    - **Faster than worklist algorithm for reachability-based data problems**
    - **The typical loop depth is low**