Lecture 2

Introduction to Data Flow Analysis

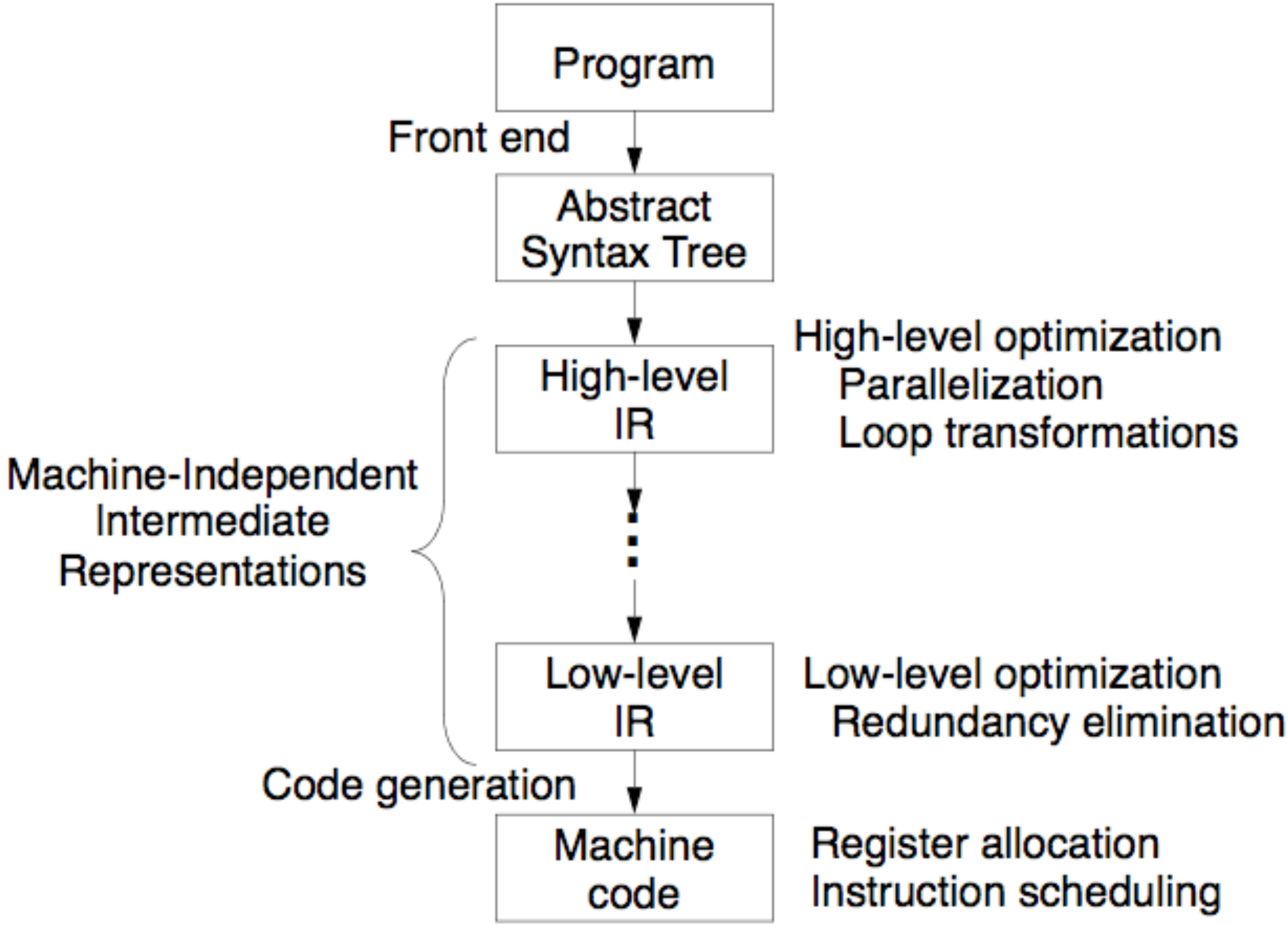
I. Introduction
II. Example: Reaching definition analysis
III. Example: Liveness analysis
IV. A General Framework
(Theory in next lecture)

Reading: Chapter 9.2

Overview of Data Flow Lectures 2-5

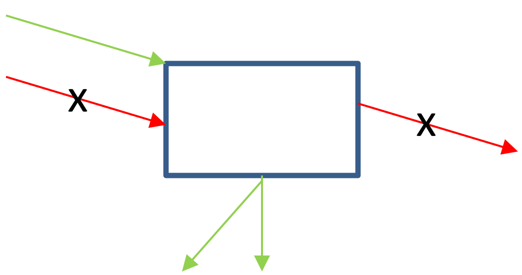
- **High-level programming languages generate a lot of redundancy**
- **Many useful optimizations independently developed originally**
  - Constant propagation
  - Common subexpressions
  - Loop invariant code motion
  - Dead code elimination
- **A common framework: Dataflow (recurrent equations, fixed-points)**
  - Theory: prove properties on the framework
  - Software engineering: implement / debug / optimize framework once
- **Plan**
  - L2: Basic examples to build intuition about dataflow
  - L3: Theory
  - L4: Optimization examples
  - L5: Partial redundancy elimination (PRE)
    Subsumes multiple optimizations by setting up 4 DataFlow problems

I. Compiler Organization



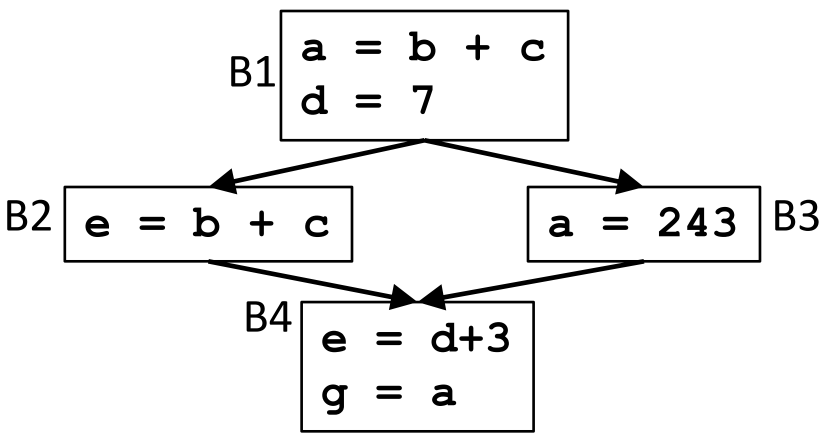
Flow Graph

- **Basic block = a maximal sequence of consecutive instructions s.t.**
  - flow of control only enters at the beginning
  - flow of control can only leave at the end
    (no halting or branching except perhaps at end of block)
- **Flow Graphs**
  - Nodes: basic blocks
  - Edges
    - *B_i --> B_j, iff B_j can follow B_i immediately in execution*



What is Data Flow Analysis?

- **Data flow analysis:**
  - Flow-sensitive: sensitive to the control flow in a function
  - intraprocedural analysis; only on pseudo variables (no aliases)
- **Examples of optimizations:**
  - Constant propagation
  - Common subexpression elimination
  - Dead code elimination



Examples of questions:

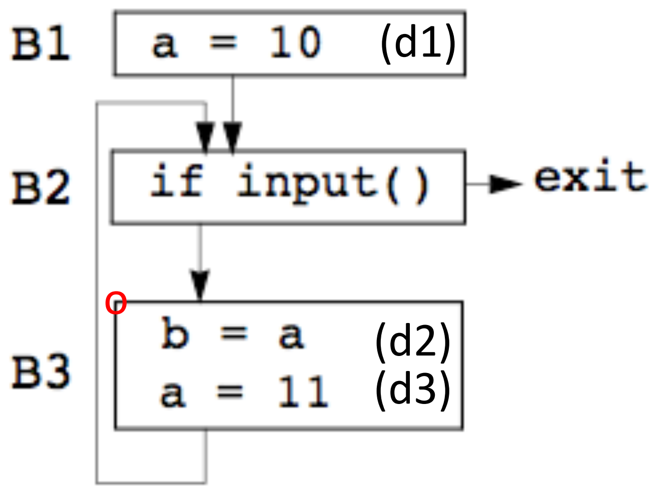
Value of x?

Which "definition" defines x?

Is the definition still meaningful (live)?

How would you code these optimizations?

Static Program vs. Dynamic Execution



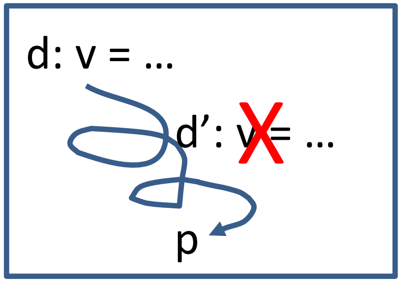
Possible executions

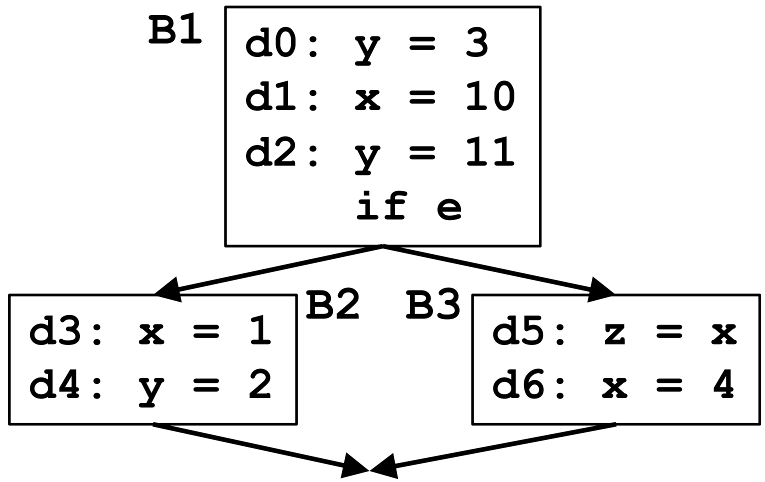
1.	2.	3.	...
B1	B1	B1	
B2	B2	B2	
	oB3	oB3	
	B2	B2	
		oB3	
		B2	

o: a point in the program

- **Statically**: Finite program
- **Dynamically**: Can have infinitely many possible execution paths
- **Example of a data flow question:**
  – Which definition defines the value used in statement "b = a"?
- **Data flow analysis abstraction:**
  – For each point in the program:
    combines information of all the instances of the same program point.
  – The definitions that can reach point o are

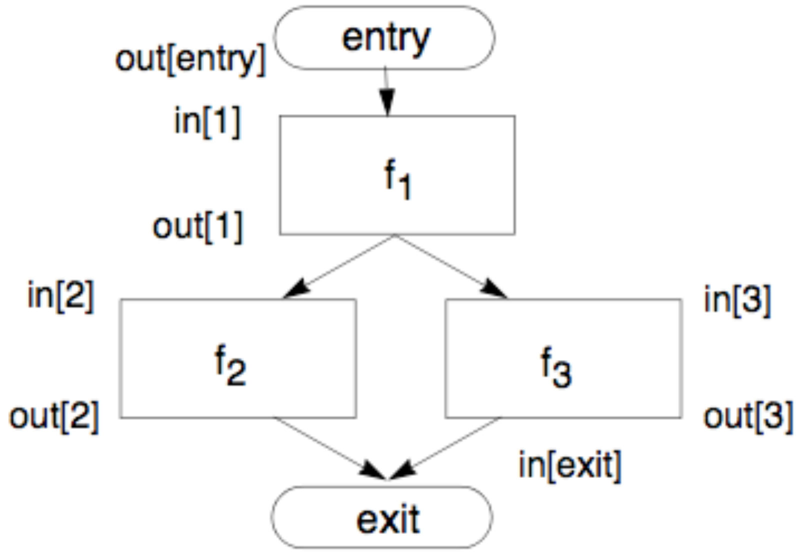
Reaching Definitions





- Every assignment is a definition
- A **definition** *d* **reaches** a point *p*
  - if **there exists** a path from the point immediately following *d* to *p*
  - such that *d* is not killed (overwritten) along that path.
- Problem statement
  - For each point in the program, determine if each definition in the program reaches the point
  - A bit vector per program point, vector-length

Data Flow Analysis Schema



- Build a flow graph (nodes = basic blocks, edges = control flow)
- Set up a set of equations between in[b] and out[b] for all basic blocks b
  - Effect of code in basic block:
    - Transfer function f_b relates in[b] and out[b], for same b
  - Effect of flow of control:
    - relates out[b_1], in[b_2] if b_1 and b_2 are adjacent
- Find a solution to the equations

Effects of a Statement

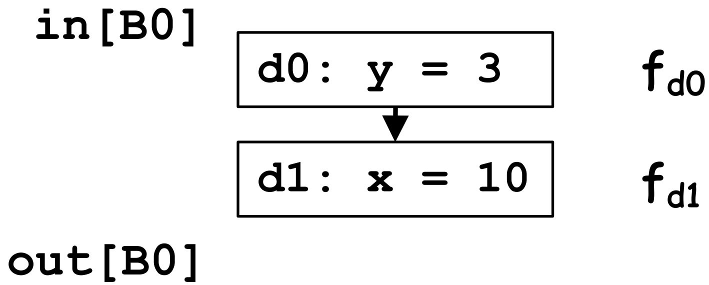
```
d0: y = 3
d1: x = 10
d2: y = 11
    if e

d3: x = 1          d5: z = x
d4: y = 2          d6: x = 4
```

Ignoring control flow

- f_s : A transfer function of a statement
  - abstracts the execution with respect to the problem of interest
- For a statement s (d: x = y + z)
  out[s] = f_s(in[s]) = Gen[s] U (in[s]-Kill[s])
  - **Gen[s]**: definitions generated: Gen[s] = {d}
  - **Propagated** definitions: in[s] - Kill[s],
    where **Kill[s]**=set of all other defs to x in the rest of program

Effects of a Basic Block



$f_B = f_{d1} \cdot f_{d0}$

- Transfer function of a statement s:
  - out[s] = f_s(in[s]) = Gen[s] U (in[s]-Kill[s])
- Transfer function of a basic block B:
  - Composition of transfer functions of statements in B
- out[B] = f_B(in[B])

  = $f_{d1} f_{d0}$(in[B])

  = Gen[d_1] U (Gen[d_0] U (in[B]-Kill[d_0]))-Kill[d_1])

  = (Gen[d_1] U (Gen[d_0] - Kill[d_1])) U in[B] - (Kill[d_0] U Kill[d_1])

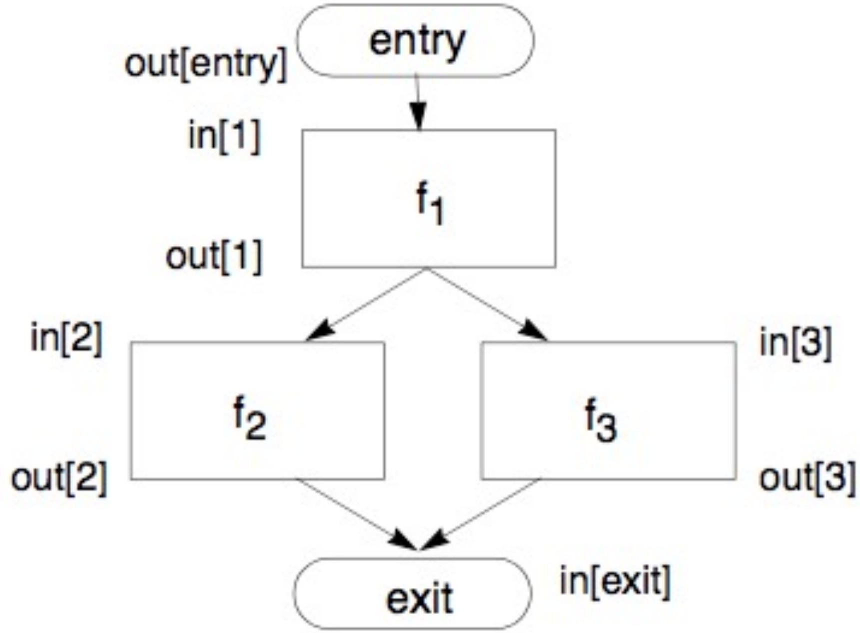
  = Gen[B] U (in[B] - Kill[B]),

  Gen[B]: (Gen[d_1] U (Gen[d_0] - Kill[d_1]))

  locally exposed definitions (available at end of bb)

  Kill[B]: Kill[d_0] U Kill[d_1] :set of definitions killed by B

Effects of the Edges (acyclic)



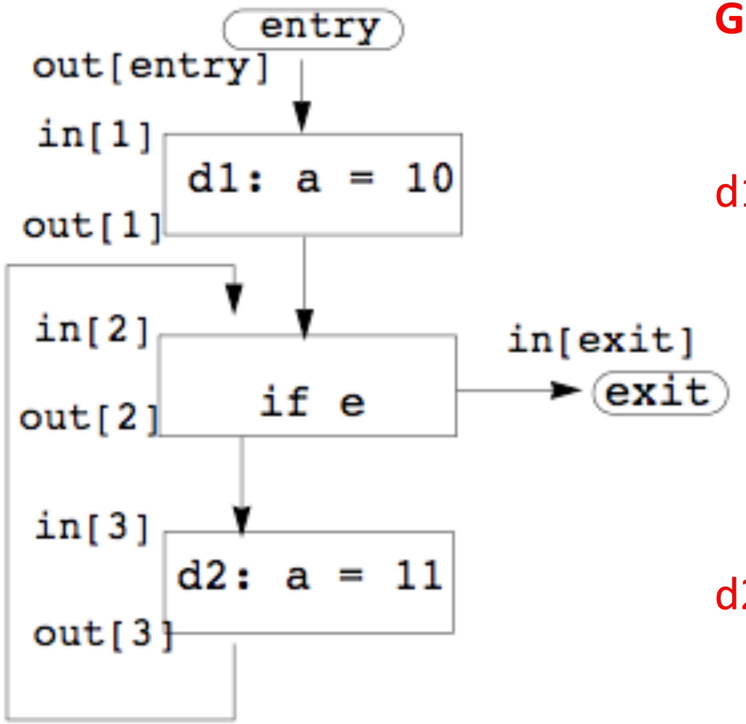
- Join node: a node with multiple predecessors
- **meet** operator ($\wedge$): U

  $in[b] = out[p_1] \cup out[p_2] \cup \ldots \cup out[p_n]$, where

  $p_1, \ldots, p_n$ are all predecessors of b

meet is a mathematical term that refers to the "meet of a semi-lattice": Not a meet of 2 control flow edges

Cyclic Graphs



Gen	Kill	Reaching Definitions
d1	d2	
		d1
		d1 d2
		d1 d2
		d1 d2
d2	d1	
		d2

- Equations still hold
  - $out[b] = f_b(in[b])$
  - $in[b] = out[p_1] \cup out[p_2] \cup \ldots \cup out[p_n]$, $p_1, \ldots, p_n$ pred.
- Find: fixed point solution

Reaching Definitions: An Iterative Algorithm

```
input: control flow graph CFG = (N, E, Entry, Exit)

// Boundary condition
   out[Entry] = ∅

// Initialization for iterative algorithm
   For each basic block B other than Entry
      out[B] = ∅

// iterate
   While (Changes to any out[] occur) {
      For each basic block B other than Entry {
         in[B] = ∪ (out[p]), for all predecessors p of B
         out[B] = f_B(in[B])    // out[B]=gen[B]∪(in[B]-kill[B])
      }
```

Quiz: Is this algorithm correct?
Does this program terminate?
Are the equations satisfied?
Include all the reaching defs?
Include any extra reaching defs?

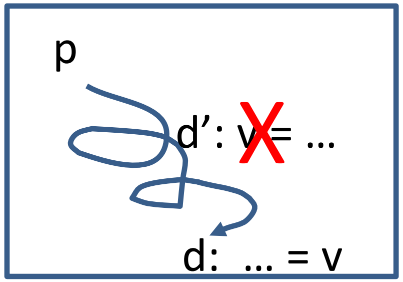
Summary of Reaching Definitions

	Reaching Definitions
Domain	Sets of definitions
Transfer function $f_b(x)$	forward: $out[b] = f_b(in[b])$ $f_b(x) = Gen_b \cup (x - Kill_b)$ Gen_b: definitions in b $Kill_b$: killed defs
Meet Operation	$in[b] = \cup\ out[predecessors]$
Boundary Condition	$out[entry] = \emptyset$
Initial interior points	$out[b] = \emptyset$

Initialization is necessary for the iterative algorithm
Not a part of the problem definition

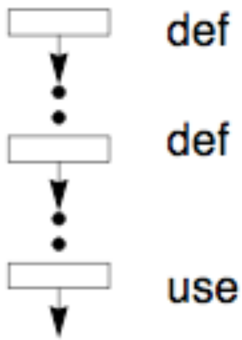
III. Live Variable Analysis

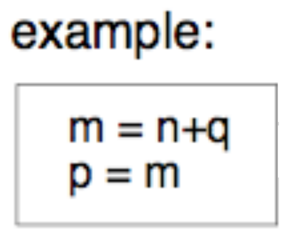
- **Definition**
  - A variable **v** is **live** at point *p* if
    - the value of **v** is used along **some** path in the flow graph starting at *p*
    - that is, **v** is not redefined along the path.
  - Otherwise, the variable is **dead**.
- **Problem statement**
  - For each basic block
    - determine if each variable is live in each basic block
  - Size of bit vector: one bit for each variable

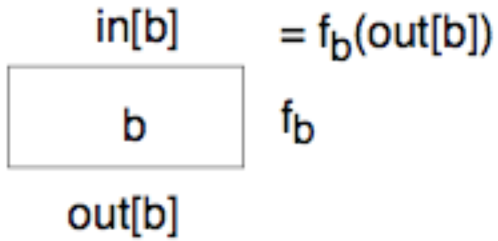


Effects of a Basic Block (Transfer Function)

- **Observation: Trace uses back to the definitions**





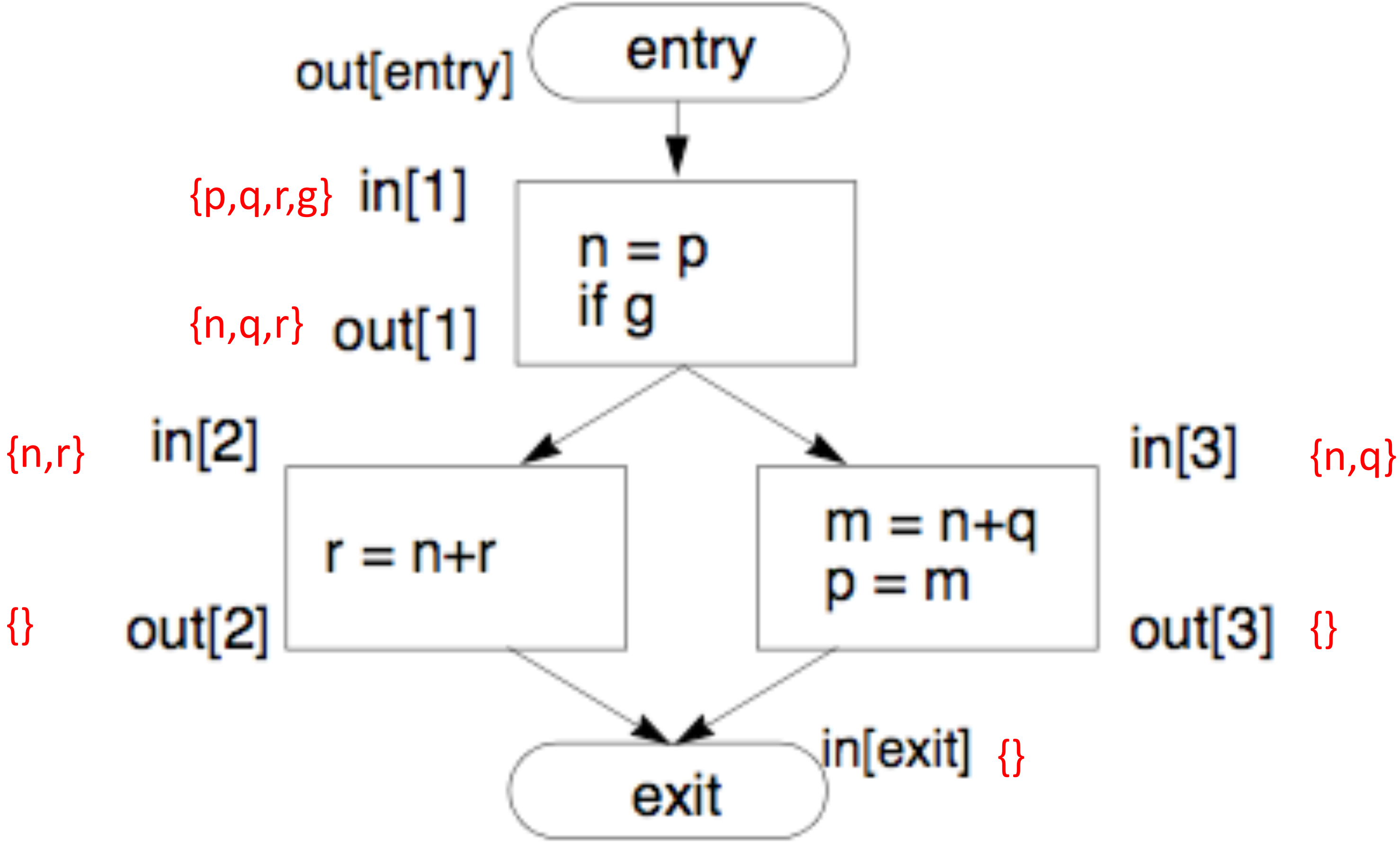


- **Direction: backward: in[b] = f_b(out[b])**
- **Transfer function** for statement s: x = y + z
  - generate live variables: Use[s] = {y, z}
  - propagate live variables: out[s] - Def[s], Def[s] = x
  - in[s] = Use[s] ∪ (out(s)-Def[s])
- **Transfer function** for basic block b:
  - in[b] = Use[b] ∪ (out(b)-Def[b])
  - Use[b], set of locally exposed uses in b, uses not covered by definitions in b
  - Def[b]= set of variables defined in b.

Across Basic Blocks

- **Meet operator ($\wedge$):**
  - $out[b] = in[s_1] \cup in[s_2] \cup \ldots \cup in[s_n]$, $s_1, \ldots, s_n$ are successors of b
- **Boundary condition:**

Example



entry
out[entry]
{p,q,r,g}
in[1]
n = p
if g
{n,q,r}
out[1]
{n,r}
in[2]
r = n+r
{}
out[2]
in[3]
{n,q}
m = n+q
p = m
out[3]
{}
in[exit]
{}
exit

Liveness: Iterative Algorithm

```
input: control flow graph CFG = (N, E, Entry, Exit)

// Boundary condition
   in[Exit] = ∅

// Initialization for iterative algorithm
   For each basic block B other than Exit
      in[B] = ∅

// iterate
   While (Changes to any in[] occur) {
      For each basic block B other than Exit {
         out[B] = ∪ (in[s]), for all successors s of B
         in[B] = f_B(out[B])    // in[B]=Use[B]∪(out[B]-Def[B])
      }
```

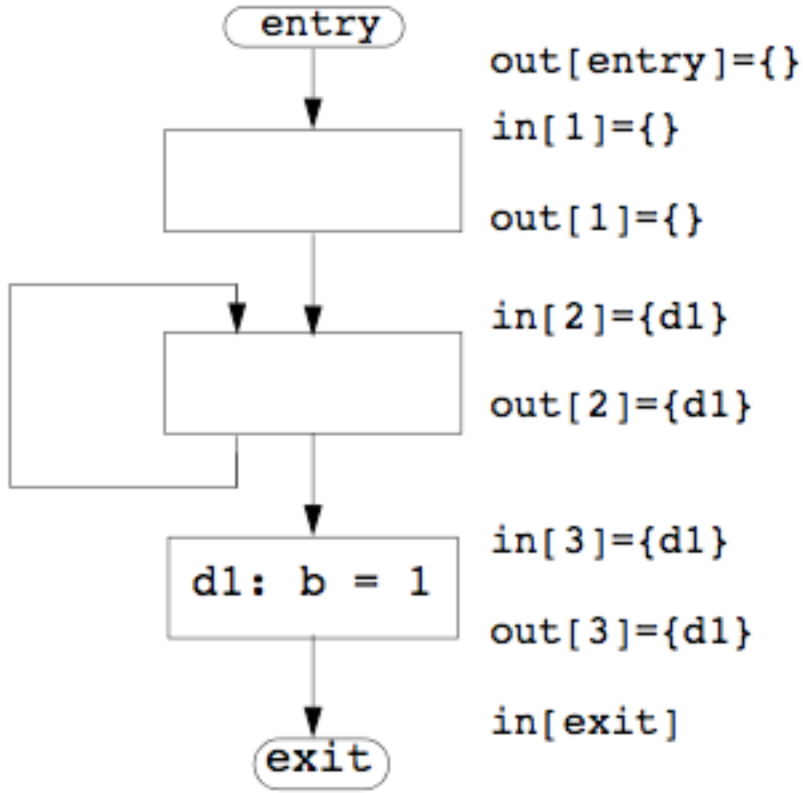
IV. Framework

	Reaching Definitions	**Live Variables**
Domain	Sets of definitions	Sets of variables
Direction	forward: $out[b] = f_b(in[b])$ $in[b] = \wedge\ out[pred(b)]$	backward: $in[b] = f_b(out[b])$ $out[b] = \wedge\ in[succ(b)]$
Transfer function	$f_b(x) = Gen_b \cup (x - Kill_b)$	$f_b(x) = Use_b \cup (x - Def_b)$
Meet Operation ($\wedge$)	$\cup$	$\cup$
Boundary Condition	$out[entry] = \emptyset$	$in[exit] = \emptyset$
Initial interior points	$out[b] = \emptyset$	$in[b] = \emptyset$

Thought Problem 1. "Must-Reach" Definitions

- **A definition D (a = b+c) must reach point P iff**
  - D appears at least once along on all paths leading to P
  - a is not redefined along any path after last appearance of D and before P
- **How do we formulate the data flow algorithm for this problem?**

Problem 2: A legal solution to (May) Reaching Def?



- Will the worklist algorithm generate this answer?

Problem 3. What are the algorithm properties?

- **Correctness**
- **Precision: how good is the answer?**
- **Convergence: will the analysis terminate?**
- **Speed: how long does it take?**