Write your answers in the space provided on the exam. If you use additional scratch paper, please turn that in as well.

Your Name: ___________________________  SUNet ID: ________________

The following is a statement of the Stanford University Honor Code:

1. The Honor Code is an undertaking of the students, individually and collectively:
   
   (a) that they will not give or receive aid in examinations; that they will not give or receive unpermitted aid in class work, in the preparation of reports, or in any other work that is to be used by the instructor as the basis of grading;
   
   (b) that they will do their share and take an active part in seeing to it that others as well as themselves uphold the spirit and letter of the Honor Code.

2. The faculty on its part manifests its confidence in the honor of its students by refraining from proctoring examinations and from taking unusual and unreasonable precautions to prevent the forms of dishonesty mentioned above. The faculty will also avoid, as far as practicable, academic procedures that create temptations to violate the Honor Code.

3. While the faculty alone has the right and obligation to set academic requirements, the students and faculty will work together to establish optimal conditions for honorable academic work.

Signature: ________________________________

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<tr>
<th>Problem</th>
<th>#1</th>
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<td>30</td>
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Problem 1. Short questions. You must \textbf{justify your answer} to each question in one to two sentences. [15 points]

1. True or false. Given a monotone dataflow analysis with only finite descending chains, if we initialize the interior points to the bottom element, running the analysis on an acyclic control flow graph (CFG) may not yield the maximum fixed point solution.

2. True or false. A monotone dataflow analysis with finite descending chains that uses reverse postorder will converge within (maximum loop depth + 2) passes.

3. Recall that in Homework 1, the \textit{Available Expression} analysis is defined with the transfer function \( \text{out}[b] = (\text{in}[b] \cup e_{\_gen[b]}) - e_{\_kill[b]} \), where \( e_{\_gen[b]} \) is the set of expressions generated by block \( b \) and \( e_{\_kill[b]} \) is the set of expressions killed by block \( b \).

   The second pass in partial redundancy elimination (PRE) computes \textit{Availability}. Its transfer function is defined as: \( \text{out}[b] = (\text{Anticipated.in}[b] \cup \text{in}[b]) - e_{\_kill[b]} \), where \( \text{Anticipated.in}[b] \) is the result from the anticipation analysis in the first pass in PRE.

   What is the relationship between the \texttt{IN/OUT} sets resulting from \textit{Available Expression} and those resulting from \textit{Availability}? Express your answer using equality or subset relations.
4. True or false. Given a program with an interference graph $G$, and the maximum node degree is $n$ (meaning every node has at most $n$ adjacent nodes). If the target machine has $n$ registers, we do not need to spill to memory in register allocation.

5. In register allocation, live ranges for the same program variable are only merged if they overlap. Suppose when we apply this technique to build the interference graph for program $P$, $n_1$ is the minimum number of registers needed to avoid spilling to memory. Suppose, for simplicity, the compiler instead merges all live ranges for the same variable, regardless of whether they overlap. Let $n_2$ be the new minimum number of registers required for the new interference graph for $P$ to avoid spilling. What is the relationship between $n_1$ and $n_2$? Express your answer as an equality or inequality.
Problem 2. Register Allocation [10 points]
Consider the following interference graph:

1. Suppose we have a machine with 2 registers. Is it possible to allocate this interference graph? If yes, demonstrate a valid coloring. [2 points]

2. If the above answer is yes, what is the maximum number of edges you can add to the interference graph for it to remain 2-colorable? If the above answer is no, how many registers do you need to allocate the interference graph without spilling? [2 points]

3. In graph theory, a clique is a subset of vertices of an undirected graph where every two distinct vertices in the clique are adjacent (i.e., an edge exists between every pair of vertices of the clique). Prove the following statement: if there exists a clique containing more than \( n \) nodes in an interference graph, the graph is not \( n \)-colorable. [3 points]

4. Consider the inverse statement. If there does not exist a clique containing more than \( n \) nodes in an interference graph, is the graph always \( n \)-colorable? If true, prove it. If not, provide a counterexample. [3 points]
Problem 3. Partial Redundancy Elimination [15 points]

Answer the following questions.

1. Show the result of running partial redundancy elimination. What’s the final optimized flow graph? You don’t need to show the intermediate steps. [12 points]

2. To model the fact that \texttt{read()} may throw an exception, we add two edges to the original control flow graph (CFG): one from B5 to exit and one from B6 to exit. Compare the optimized flow graphs after running PRE on the original CFG, and running PRE on the new CFG with the two new edges. Will the resulting CFGs be different (other than the two added edges)? Briefly explain your answer. [3 points]
Problem 4. Optimizing dynamically typed languages [30 points]

Consider a dynamically-typed language similar to JavaScript and Python. The language has three kinds of instructions:

- $x = T(...)$, where $T$ is one of \{int, float, string\}. This sets the variable $x$ to a value of type $T$.
- $x = y$, where $x$ and $y$ are variables.
- print($x$), where $x$ is a variable.

Unlike statically-typed languages like C and Java, the type of a program variable is allowed to change in this language. As an example, the program $x = \text{int}(1); x = \text{float}(3.14)$ will result in variable $x$ having an int value at first, but afterwards a float value.

Suppose we are writing an optimizing compiler for this language. Because there are special instructions for printing an integer (iPrint), printing a float (fPrint), and printing a string (sPrint), your task is to identify those print(...) instructions whose operand type can be determined at compile time. This problem has four parts: (a) Define your dataflow analysis by filling in the table below. (b) State clearly how you perform the optimization. (c) Is your dataflow analysis monotone? Explain your answer. (d) Is your dataflow analysis distributive? Explain your answer.

Part (a). Dataflow analysis specification. You may assume that each instruction is in its own basic block. [18 points]

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<thead>
<tr>
<th>Direction of your analysis (forward/backward)</th>
<th>Lattice elements and meaning</th>
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<tbody>
<tr>
<td>Meet operator</td>
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<tr>
<td>Is there a top element?</td>
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<tr>
<td>If yes, what is it?</td>
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<tr>
<td>Is there a bottom element?</td>
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<tr>
<td>If yes, what is it?</td>
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<tr>
<td>Transfer function</td>
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<td>Boundary condition</td>
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<td>Interior points</td>
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Part (b). How do you optimize the program? [4 points]

Part (c). Is your dataflow analysis monotone? Sketch a proof for it if so, or provide a counterexample if not. [4 points]

Part (d). Is your dataflow analysis distributive? Sketch a proof for it if so, or provide a counterexample if not. [4 points]