Lecture 4

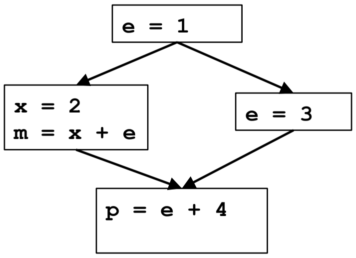
More on Data Flow: Constant Propagation, Speed, Loops

I. Constant Propagation
II. Efficiency of Data Flow Analysis
III. Algorithm to find loops

Reading: Chapter 9.4, 9.6

I. Constant Propagation/Folding

- **At every basic block boundary, for each variable v**
  - determine if v is a constant
  - if so, what is the value?



Semi-lattice Diagram

  - Finite domain?
  - Finite height?

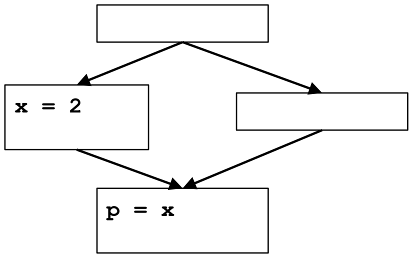
Equivalent Definition

- **Meet Operation:**

v1	v2	v1 $\wedge$ v2
undef	undef	
	c_2	
	NAC	
c_1	undef	
	c_2	
	NAC	
NAC	undef	
	c_2	
	NAC	

  - Note: undef $\wedge$ c2 = c2!

Example



Transfer Function

- **Assume a basic block has only 1 instruction**
- **Let IN[b,x], OUT[b,x]**
  - be the information for variable x at entry and exit of basic block b
- **OUT[entry, x] = undef, for all x.**
- **Non-assignment instructions: OUT[b,x] = IN[b,x]**
- **Assignment instructions: (next page)**

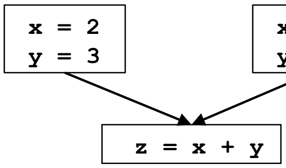
Constant Propagation (Cont.)

- **Let an assignment be of the form $x_3 = x_1 + x_2$**
  - "+" represents a generic operator
  - OUT[b,x] = IN [b,x], if $x \neq x_3$

IN[b,x_1]	IN[b,x_2]	OUT[b,x_3]
undef	undef	
	c_2	
	NAC	
c_1	undef	
	c_2	
	NAC	
NAC	undef	
	c_2	
	NAC	

- **Use:** $x \le y$ implies $f(x) \le f(y)$ to check if framework is monotone
  - $[v_1\ v_{2\ldots}] \le [v_1'\ v_2' \ldots]$, $f([v_1\ v_{2\ldots}]) \le f([v_1'\ v_2'{}_{\ldots}])$

Distributive?

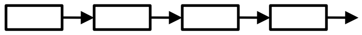


Summary of Constant Propagation

- **A useful optimization**
- **Illustrates:**
  - abstract execution
  - an infinite semi-lattice
  - a non-distributive problem

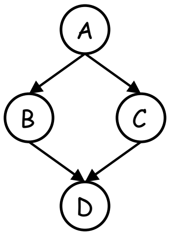
II. Efficiency of Iterative Data Flow

- **Assume forward data flow for this discussion**
- **Speed of convergence depends on the ordering of nodes**

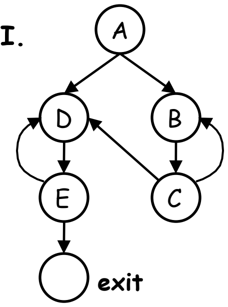


- **How about:**

I.



II.



Depth-first Ordering: Reverse Postorder

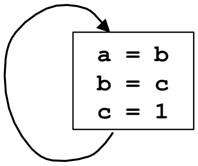
- **Preorder traversal: visit the parent before its children**
- **Postorder traversal: visit the children then the parent**
- **Preferred ordering: reverse postorder**
- **Intuitively**
  - depth first postorder visits the farthest node as early as possible
  - reverse postorder delays visiting farthest node

"Reverse Post-Order" Iterative Algorithm

```
input: control flow graph CFG = (N, E, Entry, Exit)

// Boundary condition
 OUT[Entry] = $\emptyset$

// Initialization for iterative algorithm
 For each basic block B other than Entry
    OUT[B] = $\emptyset$

// iterate
 While (changes to any OUT occur) {
   For each basic block B other than Entry in reverse post order {
      IN[B] = $\cup$ (OUT[p]), for all predecessors p of B
      OUT[B] = $f_B$(IN[B])    // OUT[B]=gen[B]$\cup$(IN[B]-kill[B])
  }
```

Consideration of Speed of Convergence

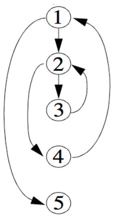
- **Does it matter if we go around the same cycle multiple times?**
- **Cycles do not make a difference:**
  - reaching definitions, liveness
- **Cycles make a difference:** constant propagation



```
a = b
b = c
c = 1
```

Speed of Convergence

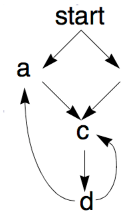
- If cycles do not add info:
  - Labeling nodes in a path by their reverse postorder rank:
    1 -> 4 -> 5 -> 7 -> 2 -> 4 ...
  - info flows down nodes of increasing reverse postorder rank in 1 pass
- Loop depth = max. # of "retreating edges" in any acyclic path
- **Maximum** # iterations in data flow algorithm = Loop depth+2
  (2 is necessary even if there are no cycles)



- Knuth's experiments show: average loop depth in real programs = 2.75

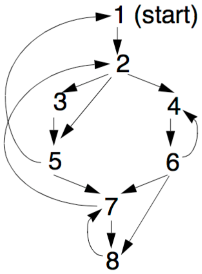
III. What is a Loop?

- **Goals:**
  - Define a loop in graph-theoretic terms (control flow graph)
  - Not sensitive to input syntax
  - A uniform treatment for all loops: DO, while, goto's
- **Informally: A "natural" loop has**
  - edges that form at least a cycle
  - a single entry point



Dominators

- Node *d* dominates node *n* in a graph (*d* dom *n*):
  - if every path from the start node to *n* goes through *d*
    - a node dominates itself



- Immediate dominance:

  d idom n : d dom n, $d \neq n$, $\neg \exists m$ s.t. d dom m and m dom n
- Immediate dominance relationships form a tree

Finding Dominators

- **Definition**
  - Node *d* dominates node *n* in a graph (*d* dom *n*) if every path from the start node to *n* goes through *d*
- **Formulated as MOP problem:**
  - node *d* lies on all possible paths reaching node *n* ⇒ *d* dom *n*
    - Direction:
    - Values:
    - Meet operator:
    - Top:
    - Bottom:
    - Boundary condition: start/exit node =
    - Finite descending chain?
    - Transfer function:
- **Speed:**
  - With reverse postorder, solution to most flow graphs (reducible flow graphs) found in 1 pass

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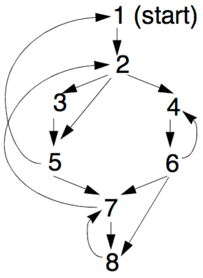
Definition of Natural Loops

- Single entry-point: ***header*** (*d*)
  - a header dominates all nodes in the loop
- A ***back edge*** (*n* → *d*) in a flow graph is
  - an edge whose destination dominates its source (*d* dom *n*)
- The **natural loop of a back edge** (*n* → *d*) is

  d **+ {nodes that can reach** ***n*** **without going through** ***d*** **}**

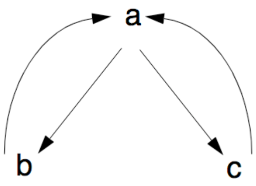
Constructing Natural Loops

- The **natural loop of a back edge** (*n* → *d*) is
  - ***d* + {nodes that can reach *n* without going through *d* }**
- Remove *d* from the flow graph, find all predecessors of *n*
- Example:



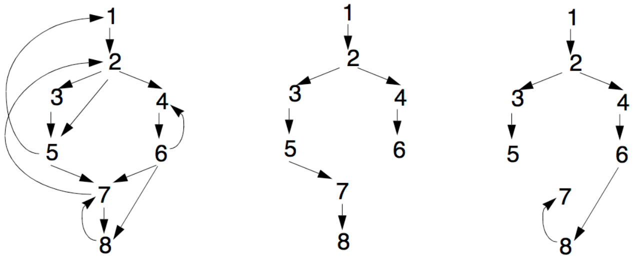
Inner Loops

- **If two loops do not have the same header:**
  - they are either disjoint, or
  - one is entirely contained (nested within) the other
    - inner loop: one that contains no other loop.
- **If two loops share the same header:**
  - Hard to tell which is the inner loop
  - Combine as one



Graph Edges

- **Depth-first spanning tree**
  - Edges traversed in a depth-first search of the flow graph form a depth-first spanning tree



- **Categorizing edges in graph**
  - Advancing edges: from ancestor to proper descendant
  - Retreating edges: from descendant to ancestor (not necessarily proper)
  - Cross edges: all other edges

Back Edges

- **Definition**
  - **Back edge**: $n \rightarrow d$, d dom n
- **Relationships between graph edges and back edges**
  - a back edge must be a retreating edge
    dominator ⇒ visit d before n, n must be a descendant of d
  - a retreating edge is not necessarily a back edge
- **Most programs (all structured code, and most GOTO programs):**
  - retreating edges = back edges

Summary

- **Constant propagation**
- **Introduced the reverse postorder iterative algorithm**
- **Define loops in graph theoretic terms**
- **Definitions and algorithms for**
    - Dominators
    - Back edges
    - Natural loops